A Study on the class of Semirings

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Abstract : *In this paper, we study the class of Right regular and Multiplicatively subidempotent semirings. Especially we have focused on the additive identity 'e' which is also multiplicative identity in both semirings.*

Keywords: *Idempotent semiring, Multiplicatively subidempotent semiring, Periodic, positively totally ordered, Rectangular band.*

I. INTRODUCTION:

Various concepts of regularity on semigroups have been investigated by R.Croisot. His studies have been presented in the book of Clifford A.H. and G.B.Preston as R.Croisot theory one of the central places in the theory is held by the left regularity. K.S.S. Nambooripad studied on the structures of regular semigroups. we introduce the notion of Right regular semiring as a generalization of regular semiring. Sen, Ghosh & Mukhopadhyay studied the congruences on inverse semirings with the commutative additive reduct and Maity improved this to the regular semirings with the set of all additive idempotents a bi semilattice.

The study of regular semigroups has yielded many interesting results. These results have applications in other branches of algebra and analysis. Section one deals with introduction. Section two contains definitions. In third section we study on the class of right regular semiring. In last section we have given some results on ordered multiplicatively subidempotent semiring.

II. PRELIMINARIES:

Definition 2.1:

A semiring S is a Right regular semiring, if S satisfies the identity $a + xa + a = a$ for all a, x in S.

Definition 2.2:

A semigroup (S, +) is rectangular band if $a = a + x + a$ for all a, x in S.
A semigroup (S, •) is rectangular band if $a = axa$ for all a, x in S.

Definition 2.3:

An element a in a semigroup (S, +) is periodic if $ma = na$ where m and n are positive integers. A semigroup (S, +) is periodic if every one of its elements is periodic.

Definition 2.4:

A semiring S is said to be idempotent if $a + a = a$ and $a^2 = a$ for all a in S.

Definition 2.5:

In a totally ordered semiring (S, +, •, ≤)

(i) (S, +, ≤) is positively totally ordered (p.t.o), if $a + x \geq a, x$ for all a, x in S
(ii) (S, •, ≤) is positively totally ordered (p.t.o), if $ax \geq a, x$ for all a, x in S.

III. CLASSES OF RIGHT REGULAR SEMIRING:

In this section, the structures of Right regular semirings with different semiring properties are given. We have also framed examples in this chapter.

Lemma 3.1: Let S be a semiring which contains the additive and multiplicative identity 'e'. Then a ($\neq$ e) in S is a Right regular element if and only if $xa = a$ for all x in S.

Proof: By hypothesis e is additive identity also multiplicative identity then $a.e = e.a = a$ and $e + a = a + e = a$ for all a, e in S
Since a in S is a Right regular element then $a + xa + a = a$ for all x in S which implies $[e + x] a + a = a$ $\Rightarrow xa + a = a \Rightarrow xa = a$

Therefore $xa = a$ for all x in S
Now we have to prove that a € S is a Right regular element
For this assume that $xa = a$ for all x in S
Let us consider $a + xa + a$
The above equation can be written as $[e + x] a + a = xa + a = xa = a$
Thus $a + xa + a = a$ for all x in S
Hence a in S is a Right regular element

Theorem 3.2: Let S be a semiring which contains an additive identity 'e' also multiplicative identity. If a ($\neq$ e) in S is a Right regular element, then $a + x = a$ for all x in S.

Proof: Given that a in S is a Right regular element then $a + xa + a = a$
By adding 'x' on both sides we get $a + xa + a + x = a + x$
This implies $a + xa + x = a + x \Rightarrow a + x (a + e) = a + x$
$\Rightarrow a + xa = a + x \Rightarrow xa = a + x$
Using above lemma $xa = a$ the above equation reduces to the form
$a + x = a$ for all x in S

Proposition 3.3: If S is a Right regular semiring with multiplicative identity '1'and (S, •) is a rectangular band, then (S, +) is periodic.

Proof: Since S is Right regular semiring then $a + xa + a = a$ for all a, x in S
This can also be written as $a^2 + axa + a^2 = a^2$
Given that (S, •) is rectangular band then $axa = a$ for all a, x in S
$\Rightarrow$ $a.1.a + axa + a.1.a = a.1.a \Rightarrow a + a + a = a \Rightarrow 3a = a$
Thus (S, +) is periodic

IV. CLASSES OF MULTIPLICATIVELY SUBIDEMPOTENT SEMIRING:

In a semiring S, an element ***a*** is Multiplicatively Subidempotent if $a + a^2 = a$. A semiring S is Multiplicatively Subidempotent if and only if each of its elements is Multiplicatively Subidempotent. Multiplicatively Subidempotent semiring plays an important role in modal logic.

Lemma 4.1: Let S be a Multiplicatively Subidempotent semiring and 'e' be additive and multiplicative identity. Then S is an idempotent semiring.

Proof: By hypothesis e is an additive identity and also multiplicative identity then $a.e = e.a = a$ and $e + a = a + e = a$ for all a, e in S
Since S is multiplicatively subidempotent $a + a^2 = a$ for all a in S$\rightarrow$ **(1)**
Equation (1) can be written as $a (e + a) = a$ which implies $a^2 = a$ $\rightarrow$ **(2)**
Thus (S, •) is a band
Adding a to both sides of equation (2) we obtain $a + a^2 = a + a$
which implies $a = a + a$ for all a in S $\rightarrow$ **(3)**
From equations (2) and (3) we conclude that S is an idempotent semiring

Example 4.2: We have framed an example by considering the set S = {a, x} for above lemma which satisfies all the conditions of lemma.

+	**a**	**x**
a	a	a
x	a	x

.	**a**	**x**
a	a	a
x	a	x

Theorem 4.3: Let S be a totally ordered multiplicatively subidempotent semiring. If S contains multiplicative identity '1' in which additive identity 'e' is also multiplicative identity and (S, +) and (S, •) are positively totally ordered then S is a mono semiring and the addition and multiplication are given by a + b = b + a = ab = ba = max (a, b).

Proof: By above lemma we have S is an idempotent semiring

Let a, b € S and $a < b$ implies $a + a \leq a + b \leq b + b \Rightarrow a \leq a + b \leq b$

Since (S, +) is p.t.o this is possible only if a + b = b = max (a, b)

Also $a < b$ implies $a^2 \leq ab \leq b^2$ which implies $a \leq ab \leq b$

Since (S, •) is p.t.o this is possible only if ab = b = max (a, b)

Example 4.4: Here (S, +) and (S, •) are p.t.o, y < x < a then a + b = b + a = ab = ba = max (a, b).

+	**a**	**x**	**y**
a	a	a	a
x	a	x	x
y	a	x	y

·	**a**	**x**	**y**
a	a	a	a
x	a	x	x
y	a	x	y

Example 4.5: Here (S, +) and (S, •) are n.t.o, a < x < y then a + b = b + a = ab = ba = min (a, b).

+	**a**	**x**	**y**
a	a	x	y
x	x	x	y
y	y	y	y

·	**a**	**x**	**y**
a	a	x	y
x	x	x	y
y	y	y	y

Note 4.6:

(i) In the above theorem if (S, +) and (S, •) are negatively totally ordered, then S is a mono semiring and the addition and multiplication are given by a + b = b + a = ab = ba = min (a, b).

(ii) In a t.o.s.r, if (S, +) is p.t.o and (S, •) is n.t.o or vice-versa. We arrive contradiction to the hypothesis that additive identity 'e' is also multiplicative identity.

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