Vibration Suppression of Vehicle-Bridge-Interaction System using Multiple Tuned Mass Dampers

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ABSTRACT :*In this study, vibration suppression of vehicle-bridge interaction system has been studied in terms of bridge dynamics. In vibration suppression, some configurations of tuned passive mass dampers have been considered. Motion equations of vehicle and absorbers are coupled with the motion equation of the bridge beam using first four mode of beam and Lagrange equations. Then the coupled equation of vehicle-bridge-absorbers (VBA) has been solved using the fourth order Raunge-Kutta algorithm. It is proved that the configuration of the absorbers that are placed on the anti-nodes of first and second mode shapes of the bridge beam is the most effective one of all others.*

KEYWORDS *-Anti-nodes,Tuned mass damper, Vehicle bridge interaction,Vibration suppression*

I. INTRODUCTION

Vibrations of bridges under moving loads are vital in engineering and have been studied over a century. Early studies of the moving load problem have been carried out by structural engineers who accepted the moving load as a moving point force. Later, with the implementation of the increased transport speed, the subject has also been examined by the automotive and railway engineers. Now the subject, as vehicle-bridge-interaction (VBI), has been studied in terms of both vehicle and bridge dynamics with the new direction such as vibration suppression of the both systems. In this field, some of analytical studies[1,2] are valuable for the dynamics of the bridge beams under different loading cases and end support conditions. When higher transportation speeds are implemented, the dynamic amplification factor (DAF) is considered as important due to the inertial effects of the moving loads in rail and other transportation systems. Vibration suppression and control of the bridge beams have been studied by [3–6]. It has been reported that using Passive Tuned Mass Dampers (PTMD), the vibration of a high-speed train bridge at 300 km/h had been reduced by 21 percent. Another application of PTMD has been reported by [4] for a thin bridge under successive moving loads. Under a periodic excitation [10] has been studied vibration reduction of a thin beam by PTMD. It has been reported in[6] that it is impossible to suppress all the vibration due to the VBI, since the excitation frequency of the moving load is not constant. For a constant velocity of the load [7], for vibration absorbers [8] , for Timoshenko beams [9], for optimized damped-absorbers [10] are the other valuable studies in this field. Plate structures under moving loads are also research interests and some FEM of such system can be found in [11]. Resonant response of a beam plate of a high-speed a railway bridge has been optimized by [12] using passive viscous dampers. Vibration suppression of a plate structure under random excitation using PTMD and its H∞ and H2 optimizations have been given in [13] and it is the early study of PTMD. It has reported that the most effective use of PTMD is to place it at a point where the mode shape is maximum at an anti-node. One of the other implementation field of the PTDM is defence science applications such as projectile barrel interaction in various type of heavy and light gun systems. For example, studies [14,15] have reported that the muzzle vibrations of barrels can be decreased at 50 percent using proper optimized PTMD.

One of the biggest challenges in PTDM use, the influence frequency of the moving vehicle is not constant and when the vehicle velocity is altered, it is changed. Secondly, there can be more than one vehicle on the bridge system with different characteristics. Thirdly, the moving mass case, taking into account the effects of mass inertia, the natural frequencies of the bridge beam are changed depending on the position of the moving load. Further, a VBI system is coupled and its linearization is difficult.

In order to suppress vibrations of the vehicle and bridge beam, in this study, the idea of the usage of the anti-nodes for the location of the PTDM in [13] has been extended, and developed such that at least the anti-nodes of the first and second mode shapes may help more effectively. Since a general vibration, response of a vibrating system can be approximated by superimposing of the effect of each vibration mode. Using this idea, in this study the vibration response of the beam has been approximated including the effect of the first four modes. Then tuned mass dampers are considered for the anti-node of the first mode at the mid-point of the bridge beam, and the anti-nodes of the second mode that are the 0.25 and 0.75 of the bridge length L. The tuning of the absorber at the mid-point is carried out using the fundamental frequency while the tunings of the absorbers at the 0.25 and 0.75 locations, the second mode frequency is used. Except this, the effects of one undamped absorber and damped absorber are also studied as a comparison. Moreover, the effect of the other position of the PTDM on beam has been analysed widely. From this perspective, this study can help both structural and automotive engineers in order to extend the service life of the bridges and predict the dynamic forces applied to the vehicle from VBI, and to achieve desired ride-safety and passenger comfort.

II. MATHEMATICAL MODELLING

In order to reduce vibrations of the vehicle and bridge due to vehicle-bridge interaction, a special system shown in Fig. 1 has been studied. The system consists of a single degree of freedom vehicle, a simply supported Euler- Bernoulli beam on which the vehicle moves with constant velocity v, and pasive vibration absorbers that are placed at 0.25, 0.5 and 0.75 of the length of the beam. Where m is the mass of the vehicle, k and c spring constant and the damping coefficient of the suspension system, while y is the vertical displacement of the vehicle body; and parameters k_i and m_i(i = 1, 2, 3), respectively, are the stiffness and mass of the absorbers at the given locations i=1, 2, and 3, as shown in Fig. 1. Symbol $y_c$is the displacement of the wheel at the contact point of the beam and $w_b(x, t)$ is the displacement of the beam at point x and time t. The displacements of the absorbers are represented by y_1, y_2, $y_3$from left to rigth.

In the formulation for the vehicle-bridge-absorbers (VBA) analysis following assumptions will be adopted:

- The bridge is modelled as a simple supported beam based on Euler-Bernoulli theory.
- The vehicle is modelled as a single DOF lumped parameter system.
- Only one vehicle is accepted moving on the bridge with constant velocity v.
- The wheel is always in contact with the bridge.
- The effect of road roughness upon vehicle and bridge dynamic is not considered during analysis in this study.

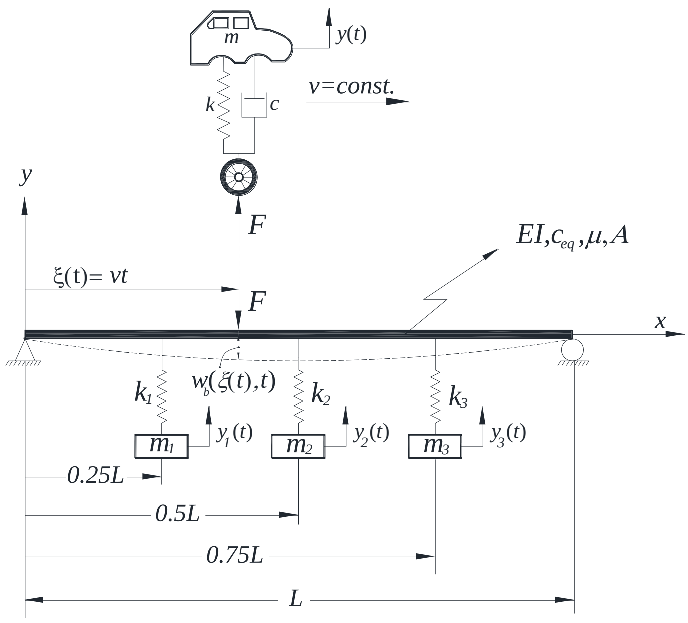

Figure 1. Model of Euler-Bernoulli Bridge beam with attached PTMDs and subjected to moving vehicle.

With all these assumptions, for the system of vehicle-bridge- absorbers (VBA) shown in Fig. 1 the kinetic and potential energy are expressed as follows, respectively:

$$E_k = \frac{1}{2}\left\{ \begin{matrix} \int_0^L \mu\left[\dot{w}_b^{\,2}(x,t)\right]dx + m\dot{y}^2(t) + \\ m_1\dot{y}_1^2(t) + m_2\dot{y}_2^2(t) + m_3\dot{y}_3^2(t) \end{matrix} \right\}, \text{(1a)}$$

$$E_p = \frac{1}{2}\left\{ \begin{matrix} \int_0^L EI\left[w_b''^2(x,t)\right]dx + k\left[y(t) - w_b(\xi(t),t)\right]^2 + \\ k_1\left[y_1(t) - w_b(L/4,t)\right]^2 + k_2\left[y_2(t) - w_b(L/2,t)\right]^2 \\ +k_3\left[y_3(t) - w_b(3L/4,t)\right]^2 \end{matrix} \right\} H(x,\xi(t)), \quad \text{(1b)}$$

whereμ and *EI*, refer to the mass of the unit length and flexural rigitidy of the bridge girder, respectively.

For the system shown in Fig.1, in order to obtain the equations of motion one can use the virtual work principle, Hamilton's principle and D'Alembert's principle. In this study, motion equation of the VBA integrated system is optained using Langrange's equations and mode expansion method. For any point *x* on the beam at time *t*the deflection function w_b *(x, t)* can be aproximated using the Galerkin's method:

$$\begin{aligned} w_b(x,t) &= \sum_{i=1}^{n} \varphi_i(x)\eta_{bi}(t), \\ \dot{w}_b(x,t) &= \sum_{i=1}^{n} \varphi_i(x)\dot{\eta}_{bi}(t), \\ w_b''(x,t) &= \sum_{i=1}^{n} \varphi_i''(x)\eta_{bi}(t), \end{aligned} \quad (2)$$

$$\varphi_i(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{i\pi x}{L}\right), \quad i = 1,2,...,n. \quad \text{(1a)}$$

whereη_{bi} is the *i-th* time dependent generalized nodal coordinate,φ_i is i-th mode shape function. The applied static axle load of the vehicle at the contact point of the wheel can be determined as:

$$f_c(x,t) = -(mgH(x-\xi(t))), \quad (3)$$

where*H(x-ξ(t))*is the Heaviside function. For the vehicle- bridge system the Rayleigh's dissipation function can be expressed as below:

$$D = \frac{1}{2}\left\{c_{eq}\dot{w}_b^{\,2}(x,t) + c[\dot{y}(t) - \dot{w}_b(\xi(t),t)]^2 H(x-\xi(t))\right\}, \text{(4)}$$

In Eq. (4) c_{eq} is the equivalent damping coefficient of bridge girder. In addition, for the given system, the Lagrangian ($L = E_k$- *Ep*) is equal to the difference of the kinetic and potential energies. In such a case, for the five independent coordinates, the Lagrange equations can be as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{p}_k(t)}\right)-\frac{\partial L}{\partial p_k(t)}+\frac{\partial R}{\partial \dot{p}_k(t)}=0\ ,\ k=1, \tag{5a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\eta}_i(t)}\right)-\frac{\partial L}{\partial \eta_i(t)}+\frac{\partial D}{\partial \dot{\eta}_i(t)}=Q_i\ ,\quad \mathrm{i}=1,2,3,4, \tag{5b}$$

With the state variables that are:

$$p(t)=\{y(t)\}^T,\ \eta(\mathrm{t})=\{\eta_1(\mathrm{t})\ \eta_2(\mathrm{t})\ \eta_3(\mathrm{t})\ \eta_4(\mathrm{t})\}^T, \tag{6}$$

And the corresponding generalized forces are:

$$Q_i=\int_0^L \varphi_i(x)\, f_c(x,t)dx,\quad i=1,2,3,4, \tag{7}$$

Using the orthogonality conditions and Galerkin approach for displacements of the beam, (Eq. (2)) the equation of motion is obtained for VBA interaction model given in Fig. 1. In this case, the equation for the acceleration of the vehicle body and absorbers are written as follows, respectively:

$$\ddot{y}=\{-c[\dot{y}(t)-\dot{w}_b(x,t)]-k[y(t)-w_b(x,t)]\}\frac{1}{m}, \tag{8a}$$

$$\ddot{y}_1=\{-k_1[y(t)-w_b(L/4,t)]\}\frac{1}{m_1}, \tag{8b}$$

$$\ddot{y}_2=\{-k_2[y(t)-w_b(L/2,t)]\}\frac{1}{m_2}, \tag{8c}$$

$$\ddot{y}_3=\{-k_3[y(t)-w_b(3L/4,t)]\}\frac{1}{m_3}, \tag{8d}$$

Dynamic equations of the bridge as the *n* second order ordinary differential equations can be expressed as follows:

$$\begin{aligned}&N_i\ddot{\eta}_i(t)+c_{eq}\varphi_i^2(x)\dot{\eta}_i(t)+\Pi_i\eta_i(t)\\&+\Lambda\varphi_i\left(\xi_1(t)\right)\begin{Bmatrix}f_c+c\left[\dot{w}_b\left(\xi(t),t\right)\Lambda_1-\dot{y}(t)\right]\\+k\left[w\left(\xi(t),t\right)\Lambda-y(t)\right]\end{Bmatrix}\\&+k_1\left[w_b(L/4,t)-y_1(t)\right]+k_2\left[w_b(L/2,t)-y_2(t)\right]\\&+k_3\left[w_b(3L/4,t)-y_3(t)\right]=0\quad i=1,2,3,4.\end{aligned} \tag{9}$$

where Λ is:

$$\Lambda=\begin{cases}1, & for\ 0\le t<t_1\\ 0 & elsewhere,\end{cases} \tag{10}$$

Where t_1 is the time when the vehicle leaves the bridge. The coefficient Λ is time dependent and is used with Eigen function $\varphi_i(\xi(t))$ in order to determine motion equation.

In Eqs. (8a), (8b-c) and (9), shows the total eight second order differential equations for the vehicle, bridge and absorbers. The first four equations are for vehicle and the absorbers and they are transferred into first order equations by [16]. In addition, the bridge dynamic is expressed by using the second order differential equations in Eq. (9). In this study, the bridge dynamic is calculated by considering the first four-vibration mode and it is represented with four differential equations of second order. These four differential equations are transferred to eight equations by [16]. In order to solve this equation system, comprising of sixteen equations, a fourth order Runge-Kutta method has been used[16].

III. NUMERICAL ANALYSIS

In this section, using four different VBA models (Model A, B, C, D) the effects of the absorbers on the vehicle and bridge dynamics have been studied extensively. System equations of motion Eq. (8a-d) for the VBA is solved by using Runge-Kutta algorithm of the fourth order, with a special m.file prepared MATLAB © environment. The integration time step size for numerical analysis during the study Δt = 0.01 s, the solution final time is taken to be $t = L / v$. In addition, the parameters used in this study for the systems VBA are presented in Table 1.

When the vehicle moves over the bridge, it forces the bridge to vibrate. At certain vehicle speeds, the bridge enters into resonance and the amount of oscillation increases significantly. The velocity of the vehicle that causes resonance is called the critical velocity v_{cr} and it is calculated using the circular natural frequencies of the beam. For a simply supported beam the natural frequencies of j^{th} vibration mode is expressed as[17]:

$$\omega_j^2=\frac{j^4\pi^4EI}{mL^4}\ (rad/s) \tag{11}$$

When Eq. (11) and the excitation frequency of a moving vehicle on the bridge are rearranged, the following is obtained[16]:

$$\alpha=\frac{\omega}{\omega_j}=\frac{\omega}{2\pi f_j}=\frac{vL}{j^2\pi}\left(\frac{m}{EI}\right)^{1/2}=\frac{v}{v_{cr}}, \tag{12}$$

Using Eq. (12), Fig. 2 shows the first three modes of vibration of the bridge beam. Frequency values of the first three modes of vibration of the bridge beam are, respectively, f_1=0.2108 Hz, f_2=0.8432 Hz, and f_3=1.8972 Hz. Using these values and Eq. (22), the critical speeds for vehicles moving on the bridge are calculated, respectively, as v_{cr1}=42.15 m/s, v_{cr2}=168.62 m/s, and v_{cr3}=252.82 m/s.

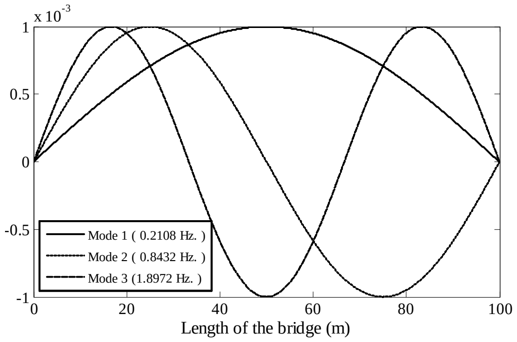

Figure2.Mode shapes of the bridge.

In order to analyze the effects of the absorbers on the dynamics of vehicles and bridges four different models of the VBA shown in Fig. 3. In the model A as shown in Fig. 3a, an undamped mass spring system is suspended from the bridge at the mid-point. The main vibration displacement of the bridge system is affected mostly from the fundemantal mode of the beam, for such reason the absorber is placed at the mid point where the maximum deflection occurs at the midpoint when the beam vibrates with only the first mode. The first natural frequency of the bridge is f_1 = 0.2108 Hz. Therefore, the absorber is tuned to resonate at the first natural frequency $f_1 = (2\pi)^{-1}\sqrt{k_1/m_1} = 0.2108$. Thus, it is expected that this absorber can reduce the vibration energy of the beam caused from the first vibration mode of the bridge. In the second model, model B, a viscous damper added to the spring mass system. The damping coefficient of the absorber was chosen as c_2=10000 Nsm^{-1}, not on purpose but to show the effect of any damping in the absorber. The critical damping value $c_{cr2} = 2m_2\omega_n = 2\sqrt{k_2 m_2} = 26476.46$ Nsm^{-1}, and the damping ratio of the damper is $\zeta = c_2/c_{cr2} = c_2/(2m_2\omega_n) = c_2/(2\sqrt{k_2 m_2}) = 0.377$ obtained using the parameters listed in Table 1. When this damper is added, the natural frequency of the absorber has reduced from 0.2108 Hz to 0.195 Hz that is the damped natural frequency, where $\omega_d = \omega_n\sqrt{1-\zeta^2} = 1.226\ rad/s = 0.195$ Hz. it is expected that adding damping may not be useful in order to reduce the vibration energy of the first mode. However, it should be known that in some case the damping could be considered in terms of design criteria. These issues are examined in detail in the following sections.

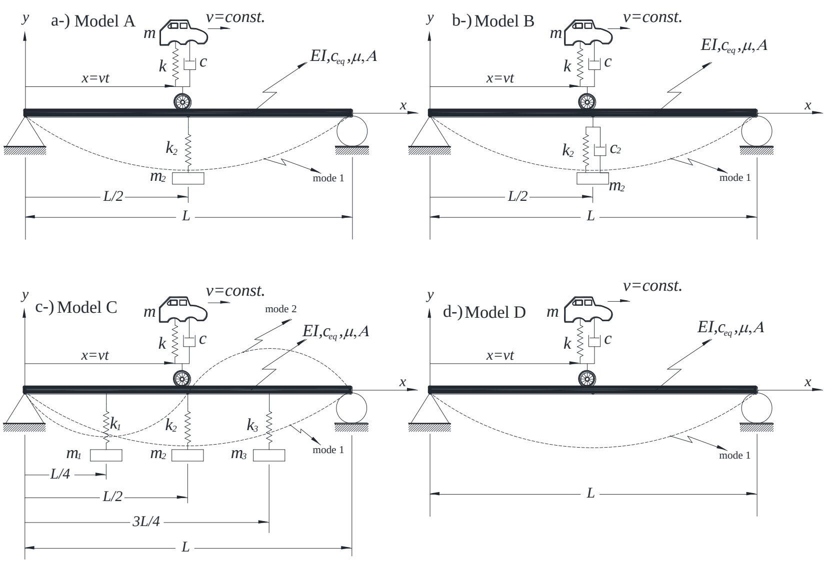

Figure 3. Model of a vehicle-bridge-absorber interaction system (a) an undamped absorber at middle of the bridge; (b) A damped absorber at middle of the bridge; (c) three undamped absorbers at 0.25L, 0.5L, and 0.75L of bridge length; (d) without absorber.

The Models A and B can reduce oscillations originating from the first mode of the bridge during vibration. Because these absorbers prepared at a middle point of maximum amplitude of the first mode of the bridge and their resonance frequency is set to the first oscillation frequency of the bridge. However, structures are not only affected from the first mode of vibration the contribution of the higher modes are also involved. Therefore, in addition to the first vibration mode for the second vibration mode of the bridge a second and a third absorber are placed at the locations 0.25 L and 0.75 L of the bridge; and they are tuned to the second natural frequency of the bridge beam. As seen from Figure 3 the peaks of the second mode shape of the beam are at these locations. In order to compare with the effects of the three previous models, the Model D has no absorber. The parameters of the bridge and the vehicle used in the study are presented in Table 1.

Table 1. Prpoperties of the vehicle and bridge.

Bridge		Vehicle parameters			
L (m)	100	m (kg)	2172	c_2 (Ns/m)	10000
E (Gpa)	207	k (kg)	85439.6	m_1(Ns/m)	10000
I (m^4)	0.174	c (kg)	2219.6	m_2(Ns/m)	10000
μ (kg/m)	20000	k_1 (N/m)	280401.59	m_3(Ns/m)	10000
c_{eq} (Ns/m)	1750	k_2 (N/m)	17525.08	k_3 (N/m)	280401.59

Based on the four different VBA models in Figure 3, Figure 4, displays the midpoint displacements w_b ($x=L/2$, t) of the bridge beam for a vehicle speed of v=90 km/h = 25 m/s that this speed is chosen so that it is applicable in transportation.

For Model D model that there is no absorber, the maximum midpoint displacement is 20.34 mm when the vehicle is at about 0.75 L, while for Model A, the maximum displacement is observed as 15.8 mm, which means a decrease of 22.32 percent when compared with no absorber case. After a damped absorber in Model B is used, the maximum midpoint displacement is 17.6 mm when the vehicle has reached to 81.2 percent of the bridge length. When a damping added to the absorber, the occurrence time of the maximum bridge midpoint displacement shifted to forward, because of the added phase difference from damping. Damping in the passive absorbers could not reduce the displacements much when compared to the undamped absorbers. For Model C, a special case when three absorbers are used, the maximum displacement of the midpoint of the bridge is 15 mm and; that means the displacement has been reduced by 26.25%. All the results are presented in Table 2.

Table 2. Comparation of the four different models according to bridge midpoint maximum displacement (m).

Model	Max. Mid-point disp.(mm)	Location of the vehicle (%)	According to the Model D, the reduction rate (%)
A	15.8	73	22.32
B	17.6	81.2	13.47
C	15	87.5	26.25
D	20.34	75	----

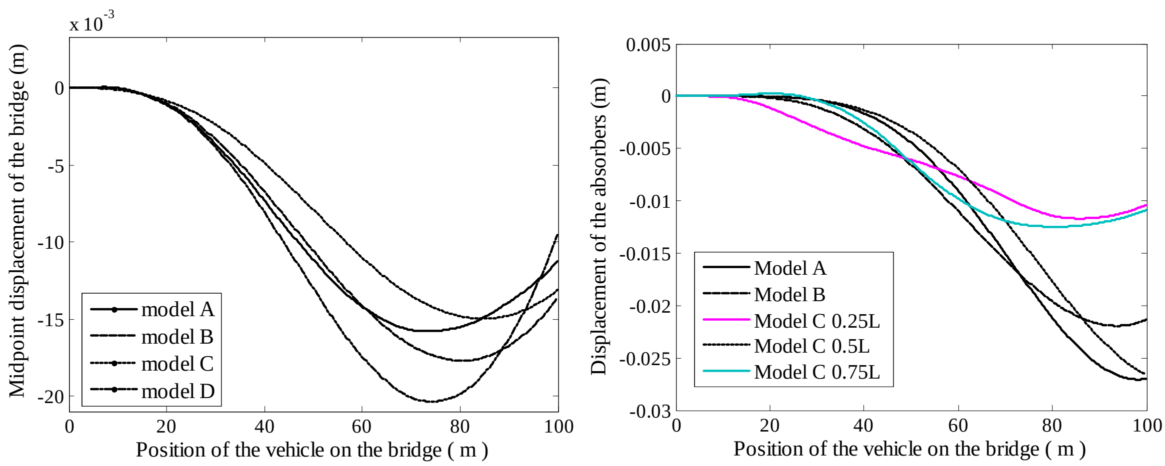

Figure 4. The effect of the absorber upon bridge midpoint dynamic response for vehicle constant velocity v=25 m/s and time step size Δt=0.01 s (a) Bridge midpoint displacement; (b) Absorber displacement.

In traditional highway or railroad bridge design, the Dynamic Amplification Factor (DAF) is considered. DAF means the ratio of the maximum dynamic displacement to the static displacement when the load is located at the midpoint of the bridge; and can be expressed as given below:

$$DAF = R_d\left(\frac{L}{2}\right)\Big/ R_s\left(\frac{L}{2}\right) \qquad (13)$$

The DAF of any bridge system can be affected by many parameters such as vehicle velocity, mass of the vehicle, natural frequency of the bridge, and road roughness, etc. considering the Models A, B, C and D, the DAFs of the different velocities of the vhecile are presented in Figure 5. As shown in Figure 5, for the Model D, that is no absorber on the bridge the maximum value of the DAF is 1.65 for the vehicle speed v = 26 m/s, while for the Model B, the maximum value of the DAF is 1.47 for the vehicle speed v = 18.5 m / s; and for the Model C the maximum value of the DAF is 1.45 for the vehicle speed v = 13.5 m / s; and for the model A the DAF=1.37 at v=12.35 m/s. Up to a speed of 31 m/s, the Model A showed the best performance, but after this speed, the Model C was the best in reducing the vibrations. Considering this situation it can genarally be accepted that velocity of the vehicle determine the behaviour of the absorbers. This is because; the forcing frequency of the VBI system is determined by the velocity of the vehicle.

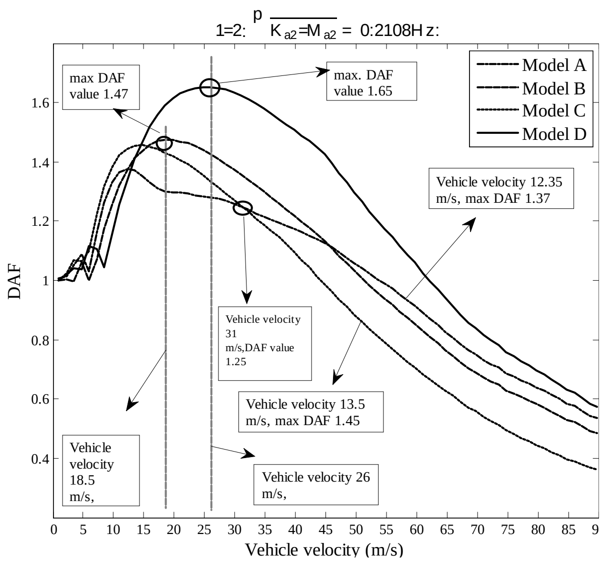

Figure 5. DAF response of the bridge mid-point for four different vehicle-bridge-absorber interaction system.

IV. CONCLUSION

In this study, in order to reduce the vibration of the vehicle that is caused by vehicle – bridge- interaction (VBI), the effects of various dynamic vibration absorbers on the bridge dynamics have been investigated. Due to the complexity of the loading in moving load problem, that the interaction is a non-linear coupled system of the two multiple degree of freedom subsystems. In the vibration of any beam system one can consider the general forced vibration response of the beam can be calculated by super positioning the mode shapes of consecutive modes at least including the first-four modes. However, in this assumption the effect of the first fundamental mode and the second mode is compromises the 80 percent of the response even in very flexible systems. In such a case, the anti-node points of the two first nodes can be used as the location of the absorbers. The anti-node is the midpoint in the first mode, and they are at 0.25L and 0.75 L in the second mode shape. From the results of the analysis, the Model C in this study, represents this situation where three absorbers have been placed on the beam at the location of the anti-nodes of the first and second modes, is the best design for the reduction of the vibrations of the bridge.

REFERENCES

[1] L. Fryba, Vibration solids and structures under moving loads, Thomas Telford House, 1999.

[2] C.I. Bajer, B. Dyniewicz, Numerical Analysis of Vibrations of Structures under Moving Inertial Load, Springer, New York, 2012. doi:10.1007/ 978-3-642-29548-5.

[3] H.C. Kwon, M.C. Kim, I.W. Lee, Vibration control of bridges under moving loads, Comput. Struct. 66 (1998) 473–480. doi:10.1016/S0045-7949(97)00087-4.

[4] J.F. Wang, C.C. Lin, B.L. Chen, Vibration suppression for high-speed railway bridges using tuned mass dampers, Int. J. Solids Struct. 40 (2003) 465–491. doi:10.1016/S0020-7683(02)00589-9.

[5] R.G. Jacquot, Optimal dynamic vibration absorbers for general beam systems, J. Sound Vib. 60 (1978) 535–542. doi:10.1016/S0022-460X(78)80090-X.

[6] Y. Kajikawa, M. Okino, Y. Matsuura, Control of traffic vibration on urban viaduct with tuned mass dampers, J. Struct. Eng. (1989).

[7] J.S. Issa, Vibration absorbers for simply supported beams subjected to constant moving loads, 226 (2012) 398–404. doi:10.1177/1464419312450652.

[8] L. Öztürk, Vibration Absorbers as Controllers, Bogazici University, 1997.

[9] M. Moghaddas, E. Esmailzadeh, R. Sedaghati, P. Khosravi, Vibration control of Timoshenko beam traversed by moving vehicle using optimized tuned mass damper, J. Vib. Control. 18 (2012) 757–773. doi:10.1177/1077546311404267.

[10] M. Tursun, E. Eskinat, H 2 Optimization of Damped-Vibration Absorbers for Suppressing Vibrations in Beams With Constrained Minimization, J. Vib. Acoust. 136 (2014) 1–14. doi:10.1115/1.4026246.

[11] İ. Esen, A new FEM procedure for transverse and longitudinal vibration analysis of thin rectangular plates subjected to a variable velocity moving load along an arbitrary trajectory, Lat. Am. J. Solids Struct. 12 (2015) 808–830.

[12] M.D. Martínez-Rodrigo, P. Museros, Optimal design of passive viscous dampers for controlling the resonant response of orthotropic plates under high-speed moving loads, J. Sound Vib. 330 (2011) 1328–1351. doi:10.1016/j.jsv.2010.10.017.

[13] Y.L. Cheung, W.O. Wong, H∞ and H2 optimizations of a dynamic vibration absorber for suppressing vibrations in plates, J. Sound Vib. 320 (2009) 29–42. doi:10.1016/j.jsv.2008.07.024.

[14] İ. Esen, M.A. Koç, Optimization of a passive vibration absorber for a barrel using the genetic algorithm, Expert Syst. Appl. 42 (2015) 894–905. doi:10.1016/j.eswa.2014.08.038.

[15] A. Littlefield, E. Kathe, R. Messier, K. Olsen, Gun barrel vibration absorber to increase accuracy, New York, 2002.

[16] M.A. Koç, İ. Esen, Modelling and analysis of vehicle-structure-road coupled interaction considering structural flexibility , vehicle parameters and road roughness †, J. Mech. Sci. Technol. 31 (2017) 1–18. doi:10.1007/s12206-017-0913-y.

[17] İ. Esen, Hareketli Yükler Altındaki Köprülü Kren Kirişlerinin Dinamik (Mukavemet) Analizi, Istanbul Technical University, 2009.