Particle Swarm Optimization Algorithm Based Window Function Design

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ABSTRACT:*The window functions used for digital filter design are used to eliminate oscillations in the FIR (Finite Impulse Response) filter design. In this work, the use of Particle Swarm Optimization (PSO) algorithm is proposed in the design of cosh window function, in which has widely used in the literature and has useful spectral parameters. The cosh window is a window function derived from the Kaiser window. It is more advantageous than the Kaiser window because there is no power series expansion in the time domain representation. The designed window function shows better ripple ratio characteristics than other window functions commonly used in the literature. The results obtained were presented in tables and figures and successful results were obtained*

KEYWORDS -*Cosh window, PSO, FIR filter*

I. INTRODUCTION

Window functions are time-domain functions that can be used to remove Gibbs' oscillations in the FIR filter design. Window functions are widely used in fields such as digital filter design, signal analysis and prediction, sound and image processing. Many window function designs with different properties are proposed in the literature. The window functions are two types, fixed and adjustable window functions, according to the variables they have.

In fixed windows, the window length (N parameter) controls the mainlobe width of the window function. Adjustable window functions with two or more variables can provide a useful amplitude spectrum. The adjustable window functions are Dolph-Chebyshev [1], Kaiser [2] and Saramaki [3] windows. Other window functions developed based on the Kaiser window are given in [4,5]. The two-parameter window functions are insufficient to control the sidelobe roll-off ratio window reduction which is window spectral parameters. In the literature, a three-parameter ultraspherical window function has been proposed instead of these functions [6-8]. The proposed window has spectral parameters such as mainlobe width, ripple ratio, null-to-null width and side-lobe pattern. PSOs, one of the heuristic computation methods, is a method derived from the movement of animals moving in swarms. With this aspect, PSO can be represented as a social interaction model [9-13].

In this work, the use of PSO in the design of the cosh window function, which is developed based on the Kaiser window and has better properties, has been proposed. The results obtained from the developed method are given by tables and graphs and the method has been shown successful.

II. WINDOW FUNCTION

Window functions are used to eliminate Gibbs' oscillations that occur in FIR filter design. In the window function design with PSO, the design of the window has been realized by an alternative method without requiring the design equations. The window functions are classified according to their spectral characteristics and compared the other windows according to these characteristics. The frequency spectrum of a window can be defined as follows.

$$W(e^{j\omega T}) = |A(\omega)| e^{j\theta(\omega)} = e^{-j\omega(N-1)T/2} W_0(e^{j\omega T}), \quad (1)$$

$$= \omega(0) + 2\sum_{n=1}^{(N-1)/2} \omega(nT)\cos\omega nT$$

where $W_0(e^{j\omega T})$ is called the amplitude function of the window, N is the window length and T is the sampling period. In the equation 1, $A(w) = |W_0 e^{jwT}|$ represents the window's magnitude and $\theta(w) = -w(N-1)T/2$ represents the window's angle. The normalized amplitude spectrum of a window can be obtained by the following equation [5].

$$(e^{jwT})| = 20log_{10}(|A(w)|/|A(w)|_{max}) \qquad (2)$$

The spectral parameters are the mainlobe width (w_M), the null-to-null width (w_N), the ripple ratio (R) and the sidelobe roll off ratio (S), which determine the window performance. From these parameters, the mainlobe width determines the width of the transition band between the pass and stop band. The ripple ratio determines the ripple at the pass and stop band, the sidelobe roll of ratio determines the distribution of the energy in the stop band.

A. *Kaiser Window*

At the discrete time, the Kaiser window can be defined as follows

$$w[n] = \begin{cases} \dfrac{I_0(\alpha_k\sqrt{1-\left(\frac{2n}{N-1}\right)^2})}{I_0(\alpha_k)} & |n| \le \dfrac{N-1}{2} \\ 0 & otherwise \end{cases} \qquad (3)$$

whereα_k is the adjustable parameter, $I_0(x)$ is the first kind zero-order Bessel function, and the power series expansion is as follows.

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2 \qquad (4)$$

B. *Cosh Window*

The cosh window, which is another two parameter window function, is obtained by writing a cosh function with similar characteristics instead of $I_0(x)$ function [5].

$$w[n] = \begin{cases} \dfrac{\cosh(\alpha_c\sqrt{1-\left(\frac{2n}{N-1}\right)^2})}{\cosh(\alpha_c)} & |n| \le \dfrac{N-1}{2} \\ 0 & otherwise \end{cases} \qquad (5)$$

III. PARTICLE SWARM OPTIMIZATION

PSO is an optimization algorithm inspired by birds and fish moving in flocks. In the PSO algorithm, each individual is called a particle and these particles constitute a swarm. Individuals in the swarm are constantly interacting with other individuals to achieve the result and with this interaction they update their current position and speed thus creating a social model. Each individual adjusts their position to the position of the best individual in the herd, taking advantage of previous experience. PSO algorithm is also an evolutionary algorithm like Genetic Algorithm (GA). However, PSO is faster than GA because there are no operators such as crossover and mutation.

The Basic PSO algorithm:

Every individual in the swarm can be a solution and every individual is represented by the dimension vector [9,10].

$$x_i = (x_{i1}, x_{i2}, x_{i3}, \ldots., x_{iD}) \in \; S \qquad (6)$$

The speed of each individual in the herd is randomly generated. Each individual has the same speed as in Equation 7.

$$v_i = (v_{i1}, v_{i2}, v_{i3}, \ldots., v_{iD}) \qquad (7)$$

The best local and global positions are determined. Here, the position of each individual is defined as follows.

$$p_i = (p_{i1}, p_{i2}, p_{i3}, \ldots., p_{iD}) \in \; S \qquad (8)$$

Each individual in the PSO adjusts its position around the individual to Pbest, global and gbest. The speed and position information of the individuals are given in equations 9 and 10.

$$v_i^{(t+1)} = v_i^{(t)} + c_1 r_{i1} * \left(pbest_i^{(t)} - x_i^{(t)}\right) + c_2 r_{i2} * (gbest - x_i^{(t)}) \qquad (9)$$

$$^{+1)} = x_i^{(t)} + v_i^{(t+1)}, \qquad i = 1, \ldots \ldots, P \qquad (10)$$

Here c1 and c2 are two social and cognitive acceleration parameters. r1 and r2 are random numbers between [0,1]. The general PSO algorithm is as given below.

Algorithm 1. Particle Swarm optimization algorithm

Set the initial value of P (swarm size) and c_1,c_2 (acceleration constants)

Set t=0

Generate $x_i^{(t)}$ and $v_i^{(t)}$ randomly

Evaluate the fitness function $f(x_i^{(t)})$

Set $gbest^{(t)}$ (where *gbest* is the best glocal solution)

Set $pbest_i^{(t)}$ (where *pbest*is the best local solution)

repeat

$$v_i^{(t+1)} = v_i^{(t)} + c_1 r_{i1} * \left(pbest_i^{(t)} - x_i^{(t)}\right)$$
$$+ c_2 r_{i2} * (gbest - x_i^{(t)})$$

$$x_i^{(t+1)} = x_i^{(t)} + v_i^{(t+1)}, \qquad i = 1, \dots \dots, P$$

Evaluate the fitness function $f(x_i^{(t+1)})$

iff $(x_i^{(t+1)}) \leq f(pbest_i^{(t)})$***then***

$$pbest_i^{(t+1)} = x_i^{(t+1)}$$

else

$$pbest_i^{(t+1)} = pbest_i^{(t)}$$

end if

if$x_i^{(t+1)} \leq f(gbest^{(t)})$***then***

$$gbest^{(t+1)} = x_i^{(t+1)}$$

else

$$gbest^{(t+1)} = gbest^{(t)}$$

end if

t=*t*+1

until termination criteria are satisfied.

Show the best particle

IV. DESIGN RESULTS

The cosh window function designed with PSO for N = 15 and alphac= 1.73 is given in Figure 1 and the graphical data for graph is given in Table 1. Error variation is given in Figure 2.

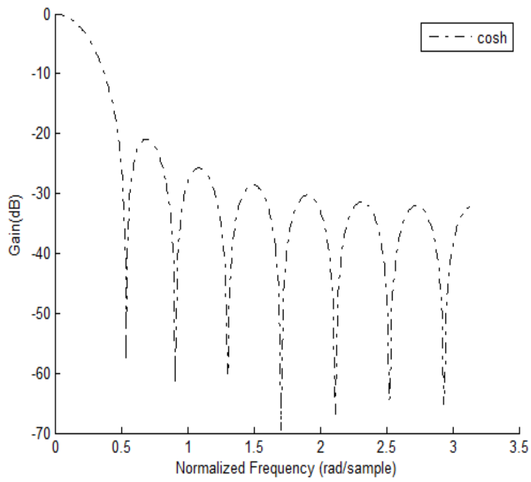

Figure 1: Amplitude response of cosh window for N=15 and alphac=1.73

Table 1: Obtained Results

Cosh Window	WR	S	R
N=15, alphac=1.73	0.478	11.23	-20.97

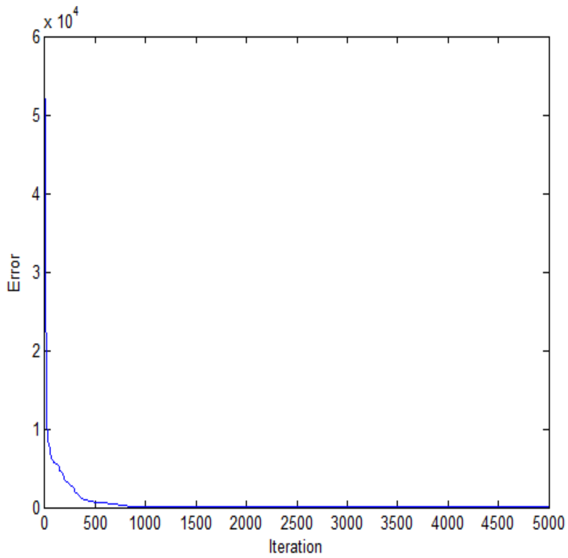

Figure 2: Error variation

V. CONCLUSION

In this study, the cosh window function is designed by using PSO algorithm. The success of the developed method has been made for the cosh window which is preferred in the literature in terms of its properties. The obtained results are shown in graphical form. The results showed that the method used was successful in response to the window amplitude.

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