Fault Diagnosis of Partially Observed Discrete Event Systems in Petri Nets with Variable Elimination Method

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ABSTRACT : *The demand to understand complex systems for those involved is high. Systems are independentlyattached instead of a combination of them, where some subsystems are discrete event dynamic. The merging of Petri nets offers a relatively mature body and extremely promising for dealing with complex discrete event dynamic systems. This paper presents a fault diagnosis method based on variable elimination for partially observed discrete event systems in a Petri net, where the faults are in unobservable transitions. The variable elimination method is Fourier- Motzkin witch consists of eliminating the desired variable from a set of inequalities. The fault diagnosis method used is offline and online. In the offline diagnosis, we will obtain two sets of inequalities from the Petri net state equation and apply the Integer Fourier-Motzkin Elimination method to eliminate all variables corresponding to unobservable transitions in the inequalities. Then, from online diagnosis, the reduced set of inequalities obtains the state diagnosis by observing and verifying the sequence of events after each occurrence of an observable event.*

KEYWORDS -*Discrete Event System, Fault Diagnosis, Partially Observable, Petri Nets.*

I. INTRODUCTION

Discrete event systems (DESs) are those where the state sets are discrete and whose evolution occurs through events and not through time [3]. These systems perceive events in the external world from the reception of stimulation events. Examples of events are tasks start and the completion and the reception of a message in a communication system. The occurrence of an event causes, in general, an internal change in the system, which may or may not be manifest to an outside observer. Nowadays, DESs are in manufacturing, robot, transportation system, and many others. Automation systems are subject to the occurrence of faults that can change their normal behavior.

Faults refer to a total or partial decrease in the performance capacity of a component, equipment, process, or system to fulfill a function during a period. Faults are events that cannot, by their very nature, be eliminated in real life [8], and the systems that contain faults behave differently than expected. However, it does not necessarily suspend the system; for example, in manufacturing systems, an undiagnosed fault can lead to a degradation of the indicators of the overall effectiveness of equipment (availability, efficiency, and quality) [10]. The need for adequate procedures to detect faults is quite evident, considering its consequences and impacts in these areas.

Thus, the study on fault diagnosis in DES is suggested and consists of demonstrating the occurrence of faults based on observing events generated by the system [3]-[12]. Fault diagnosis is crucial in most industrial systems when maintaining equipment safety.

This research direction seeks to find efficient and reliable ways to detect occurrences' faults and their isolation. However, as systems grow in complexity and size, automatically obtaining accurate and detailed dependency models to capture the different characteristics of their behavior becomes challenging. The vast amount of work in the literature has treated the problem as a significant challenge for systems diagnosis.

Fault diagnosis is detecting an abnormality in the system's behavior. It consists of checking the system's behavior after an observable occurrence [9]. In diagnosing faults in partially observed DES, the defects to be analyzed are unobserved events, that is, events whose occurrences cannot be recorded or detected by sensors.

Lin [6] introduced the capacity to diagnose the occurrence of a fault in the systems concept, which inserted the problem of fault diagnosis into the context of DES. Soon after, Sampath et al. [10] presented necessary and sufficient conditions for fault diagnosis of DESs and proposed the construction of a diagnostic automaton that allows both inferences about the ability to diagnose the faults present in the system when being used to perform real-time fault diagnosis.

Over past decades, two formalisms have been used to help with the problem of fault diagnosis in partially observable discrete event systems, Automata and Petri nets. Automata models guide creating a diagnoser automaton to check whether the occurrences of unobservable events are possible by observing words with finite lengths [5]. Although automata models are suitable for DESs, the system size would limit their implementation. As these models specify all the possible states, it would result in large models. Thus, Petri nets are more appropriate for addressing faults diagnosis, given their excellence in graphical structure.

We will use the Petri net, in this paper as they offer significant advantages due to their graphic, mathematical representation ethics and ability to analyze, control, validate and implement in different systems, especially discrete event systems [7]. The problem of diagnosis is considered through the modeling of faults as unobservable transitions and an online diagnosis that observes sequences of observable events and issues a decision on the occurrence of fault based on the solution of problems [2]-[4].

This paper addresses the problem of diagnosis inspired by the work of Al-Ajeli and Parker [1], they use acrylic Petri nets where observable, unobservable, and faulty transitions are in the same event and introduces the elimination method, Integer Fourier-Motzkin Elimination (IFME) originated from Fourier-Motzkin Elimination (FME).

FME is an extension of Gaussian elimination methods used in equations, whereas the FME is for inequalities. IFME method eliminates all the proposed variables separately. It will be helpful to drop all the variables from unobservable transitions and construct two sets of inequalities [1]-[2]. After eliminating the variables concerned with unobservable transition, check the variable reports to observable events for a given sequence of observed events and find the state of the diagnosis.

The paper is organized as follows. Section II introduces the concepts of the Petri nets and the variable elimination method. Section III displays the proposed approach obtaining sets of inequalities from the equation state associated with the Petri net through offline diagnosis. In Section IV, we get the main result through the online diagnosis, compute the states, and finally, the conclusion in section V.

II. PRELIMINARIES

2.1 Petri Nets

A Petri net (PN) is $N = (P, T, Pre, Post)$ [7] where $P = \{p_1, ..., p_u\}$ is a set of places, $T = \{t_1, ..., t_v\}$ is a set of transition, *pre, and Post condition* : $P \times T \rightarrow N$. The function M represents a state of a Petri net $M: P \rightarrow N$, which captures the number of chips in each place; (N, M_0) denotes a Petri net with the initial marking M_0. If an M is accessible from M_0 through a sequence transitions $\sigma = t_1 ...t_k$, then there is a vector x such that the following states equation of state is satisfied:

$$M = M_0 + Ax \geq 0.(1)$$

Where A = $[a_{ij}]$ is the m×n matrix called the incidence matrix, where $aij = post(p,t) - pre(p,t)$ being the weight of the transitions arc to place, $x = [x_i, ..., x_n]^T T \in N$ is the firing count vector, where x_i represents the number of occurrences of the transition $t_i \in T$.

Given a matrix with ten rows representing places and thirteen column representing transitions, to get the post condition and precondition , we have to consider the weight of the arc connecting the places and transitions to be 1. The Post condition is obtained when the places receives the inputs of the transitions or when transition input to place $(p \leftarrow t$ or $t \rightarrow p)$ where (p_1), (t_1, p_2), (t_2, p_3), (t_3, p_4), (t_4, p_5), (t_5, p_6), (t_6, p_5), (t_7, p_7), (t_8, p_8), (t_9, p_5), (t_{10}, p_9),(t_{11}, p_{10}), (t_{12}, p_5). For the precondition the places input to transitions or the transitions receive from places $(p \rightarrow t$ or p $t \leftarrow p)$ where (p_1, t_1) (p_2, t_2), (p_2, t_5), (p_2, t_7), (p_2, t_{10}), (p_3, t_3), (p_4, t_4), (p_4, t_{13}) , (p_5, t_{13}), (p_6, t_6) , (p_7, t_8), (p_8, t_9), (p_8, t_{13}) , (p_9, t_{11}), (p_{10}, t_{12}), (p_{10}, t_{13}).

Example 1. Let us take in consideration the Petri nets in the Fig. 1, where $P = \{p_1, ..., p_{10}\}$, $T = \{t_1, ..., t_{13}\}$, M_0 = [1000000000], $T_o= \{t_1, t_3, t_4, t_5, t_9, t_{10}, t_{11,}t_{12}, t_{13}, …, t_v\}$, and $T_u= \{t_2, t_6, t_{7,} t_8, ... t_n\}$. The observable transitions T_o are represented by solid rectangles, while the empty rectangles represents transitions associated with unobservable transitions T_u, There are two faultstransition $T^1_f = \{t_6\}$ and $T^2_f =$

{t_8}, and they can be written as c: $=x_6 \leq 0$ and c: $=x_8 \leq 0$; their negation as ¬c: = $x_6>0$ or ¬c: = $-x_6 \leq -1$; ¬c: = $x_8>0$ or ¬c: = $-x_8 \leq -1$.

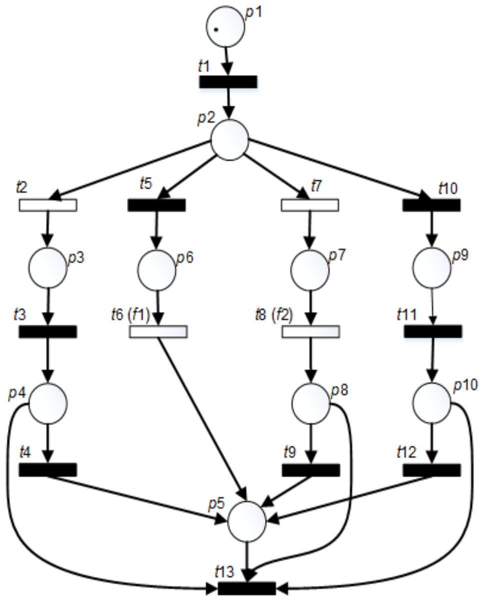

Figure 1.Example of Petri net.

The state equation (1) for the Petri net is given by:

$$M = \begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\\x_8\\x_9\\x_{10}\\x_{11}\\x_{12}\\x_{13}\end{bmatrix} \geq 0$$

where

$$\text{Post} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&1&0&0&1&0&0&1&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0 \end{bmatrix}$$

$$\text{Pre} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&1&0&1&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&1 \end{bmatrix}$$

The sequence of events $s=\{t_1\}$ will produce two sequences $\sigma_1 = t_1t_2$ and $\sigma_2 = t_1t_7t_8$;one has the second type of fault, and another does not; hence we are not sure if the the second fault has happened but we are sure the first did not in the sequences, as for the sequence s = {t_1, t_5} we are not sure the first fault has happen, and sure the second does not.

Let us assume the sequence is now $s=\{t_1, t_{10}, t_{11}, t_{12}\}$; this sequence produces an observable sequence of events. We are sure that the fault has not occurred because there is no other sequence of events that contains the fault with the same sequence of observable events during the observation, the system has a normal behavior. Suppose the sequence is $s=\{t_1, t_9\}$; it produces a sequence of the observable transition and there is a fault T^2_f, in this case we are sure that the fault has occurred.

2.2 The Integer Fourier- Motzkin Elimination Method

The elimination of a variable in an inequality system $-Ax \leq b$ where $A \in R^{m\times n}$, $b \in R^m$, and $x = (x_1, x_2, ..., x_n) \in R^n$ [1]-[11]. We can get all the elements in the last column of $-A$ as 0, +1, or −1 by multiplying each inequality by a positive scalar and then the set of inequalities can be rewritten as if.

$$\begin{aligned} a'_i x' &\leq b_i,\ i = 1,\ldots,m_1 \\ a'_j x' - x_n &\leq b_j,\ j = m_1+1,\ldots,m_2 \\ a'_k x' + x_n &\leq b_k,\ k = m_2+1,\ldots,m. \end{aligned} \quad (2)$$

Where $x' = \{x_1, x_2, ... x_{n-1}\}$. Assume that $L = max\ (a'_j x' - b_j, j = m1+1, ..., m2)$ and $U = min\ (b_k - a_k x', k = m2+1, ..., m)$. Since the last two lines (2) are equivalent to $L \leq x_n \leq U$, then the variable $x_n$can be eliminated obtaining:

$$\begin{aligned} a'_j x' &\leq b_i,\quad i = 1,\ldots,m_1 \\ a'_j x' - x_n &\leq b_k - a'_k x',\ j = m_1,\ldots,m_2 \\ k &= m_2+1,\ldots,m. \end{aligned} \quad (3)$$

By repeating this process, we can successively eliminate unwanted variables.

III. THE PROPOSED APPROACH FOR FAULT DIAGNOSIS

The Petri nets are divided into observable transitions T_o and unobservable transitions T_u. All the faults are in unobservable transitions; the set of transitions that are modelling occurrences of faults is T_f. A system can contain different types of faults implies $T^i_f = \{T^1_f ... T^r_f\}$.The process of faults diagnosis can be divided into two steps: offline step and online step.

The offline diagnoses will be helpful for online diagnosis on systems for Partially Observed Discrete Events (DES) modeled Petri nets. The notion of using the IFME for fault diagnosis in DES modeled by Petri nets is introduced. The online diagnoser will help us compute the diagnosis state through the reduced sets of the inequalities. The offline method begins with the eq. (1). Since each marking M is non-negative, i.e., $M \geq 0$, the equation is rewritten as:

$$-Ax \leq M_o. \quad (4)$$

As faults transitions are associated with c and ¬c, these set of inequalities can be defined as:

$$c := \sum_{t_i \in T_f} x_i \leq 0 \ \ and\ \ \neg c := \sum_{t_i \in T_f} x_i > 0. \quad (5)$$

The I represents the set of inequalities; it will add separately the inequalities c and ¬c to have $I \cup \{c\}$ and $I \cup \{\neg c\}$ and eliminate all the variables corresponding to the set of unobservable transitions by applying the IFME. Below we are computing the offline algorithm used for the diagnose and inequalities obtained from the eq. (1) associated with constraints:

Algorithm 1 Offline Diagnoser

Input:

• *N= (P, T, pre, post, M_0):* Petri net model.

• T_o, T_u, T_f: Sets of observable transition, unobservable and faulty transitions respectively.

Output:

Set of inequalities with variables that quantify the occurrences of observable events being R and R'.

1: Determine the equation of state of the Petri net $M = M0 + Ax$.

2: Form the set I of inequalities $-Ax \leq M_0$ and $x_i \leq 0$.

3: Determine T_f = {t ∈ T: ' (t) = f} and then, the constraints c and ¬c that models the occurrence of faults.

4: Form the sets I ∪ {c} and I ∪ {¬c}.

Table 1: The sets of inequalities and its constraints

$I \cup \{C\}$	$I \cup \{\neg c\}$
$x_1 \leq 1$	$x_1 \leq 1$
$-x_1+x_2+x_5+x_7+x_{10} \leq 0$	$-x_1+x_2+x_5+x_7+x_{10} \leq 0$
$-x_2+x_3 \leq 0$	$-x_2+x_3 \leq 0$
$-x_3+x_4+x_{13} \leq 0$	$-x_3+x_4+x_{13} \leq 0$
$-x_4-x_6-x_9-x_{12}+x_{13} \leq 0$	$-x_4-x_6-x_9-x_{12}+x_{13} \leq 0$
$-x_5+x_6 \leq 0$	$-x_5+x_6 \leq 0$
$-x_7+x_8 \leq 0$	$-x_7+x_8 \leq 0$
$-x_8+x_9+x_{13} \leq 0$	$-x_8+x_9+x_{13} \leq 0$
$-x_{10}+x_{11} \leq 0$	$-x_{10}+x_{11} \leq 0$
$-x_{11}+x_{12}+x_{13} \leq 0$	$-x_{11}+x_{12}+x_{13} \leq 0$

$-xi \le 0 \; i \in \{2, 6, 7, 8\}$	$-xi \le 0 \; i \in \{2, 6, 7, 8\}$
$x_6 \le 0$	$-x_6 \le -1$

$I \cup \{C\}$	$I \cup \{\neg c\}$
$x_1 \le 1$	$x_1 \le 1$
$-x_1+x_2+x_5+x_7+x_{10} \le 0$	$-x_1+x_2+x_5+x_7+x_{10} \le 0$
$-x_2+x_3 \le 0$	$-x_2+x_3 \le 0$
$-x_3+x_4+x_{13} \le 0$	$-x_3+x_4+x_{13} \le 0$
$-x_4-x_6-x_9-x_{12}+x_{13} \le 0$	$-x_4-x_6-x_9-x_{12}+x_{13} \le 0$
$-x_5+x_6 \le 0$	$-x_5+x_6 \le 0$
$-x_7+x_8 \le 0$	$-x_7+x_8 \le 0$
$-x_8+x_9+x_{13} \le 0$	$-x_8+x_9+x_{13} \le 0$
$-x_{10}+x_{11} \le 0$	$-x_{10}+x_{11} \le 0$
$-x_{11}+x_{12}+x_{13} \le 0$	$-x_{11}+x_{12}+x_{13} \le 0$
$-xi \le 0 \; i \in \{2, 6, 7, 8\}$	$-xi \le 0 \; i \in \{2, 6, 7, 8\}$
$x_8 \le 0$	$-x_8 \le -1$

IV. MAIN RESULTS

In this section, we will compute the diagnosed state of the given Petri net from the reduced set R and R' created by eliminating every variable corresponding to unobservable transition in the set T_u. The advantage of using R and R' is that since all variables relate to observable events, we can check that for a given sequence σ if the projection to observable events satisfies R and R'. Below is the reduced set of inequalities to help computing the diagnosis state.

Table 2: Reduced Sets of inequalities

R_1	R'_1
$x_1 \le 1$	$x_1 \le 1$
$-x_3+x_4+x_{13} \le 0$	$-x_3+x_4+x_{13} \le 0$
$-x_{10}+x_{11} \le 0$	$-x_{10}+x_{11} \le 0$
$-x_{11}+x_{12}+x_{13} \le 0$	$-x_{11}+x_{12}+x_{13} \le 0$
$-x_4-x_9-x_{12}+x_{13} \le 0$	$-x_5 \le 0$
$-x_4-x_5-x_9-x_{12}+x_{13} \le 0$	$-x_5 \le -1$
$-x_5 \le 0$	$-x_4-x_5-x_9-x_{12}+x_{13} \le 0$
$-x_1+x_3+x_5+x_{10} \le 0$	$-x_1+x_3+x_5+x_{10} \le 0$
$-x_1+x_3+x_5+x_9+x_{10}+x_{13} \le 0$	$-x_1+x_3+x_5+x_9+x_{10}+x_{13} \le 0$
$-x_1+x_3+x_5+x_{10} \le 0$	$-x_1+x_3+x_5+x_{10} \le 0$
$-x_1+x_5+x_{10} \le 0$	$-x_1+x_5+x_{10} \le 0$
$-x_1+x_5+x_9+x_{10}+x_{13} \le 0$	$-x_1+x_5+x_9+x_{10}+x_{13} \le 0$
$-x_1+x_5+x_{10} \le 0$	$-x_1+x_5+x_{10} \le 0$

R_2	R'_2
$x_1 \le 1$	$x_1 \le 1$
$-x_3+x_4+x_{13} \le 0$	$-x_3+x_4+x_{13} \le 0$
$-x_{10}+x_{11} \le 0$	$-x_{10}+x_{11} \le 0$
$-x_{11}+x_{12}+x_{13} \le 0$	$-x_{11}+x_{12}+x_{13} \le 0$
$x_9+x_{13} \le 0$	$-x_4-x_5-x_9-x_{12}+x_{13} \le 0$
$-x_4-x_5-x_9-x_{12}+x_{13} \le 0$	$-x_5 \le 0$
$-x_5 \le 0$	$-x_1+x_3+x_5+x_{10} \le 0$
$-x_1+x_3+x_5+x_{10} \le 0$	$-x_1+x_3+x_5+x_9+x_{10}+x_{13} \le 0$
$-x_1+x_3+x_5+x_9+x_{10}+x_{13} \le 0$	$-x_1+x_3+x_5+x_{10} \le 0$
$-x_1+x_3+x_5+x_{10} \le 0$	$x_1+x_3+x_5+x_{10} \le -1$
$-x_1+x_5+x_{10} \le 0$	$-x_1+x_5+x_{10} \le 0$
$-x_1+x_5+x_9+x_{10}+x_{13} \le 0$	$-x_1+x_5+x_9+x_{10}+x_{13} \le 0$
$-x_1+x_5+x_{10} \le 0$	$-x_1+x_5+x_{10} \le 0$
	$-x_1+x_5+x_{10} \le -1$

A diagnoser is a function $\Delta: T^{*}_{o} \times 2^{T}{}_{f,}$ which indicates if the fault has not occurred, if the fault has occurred and if if there is doubt that a fault has occurred. The diagnose Δ *(s, T_f)* will be a mapping Δ *(s, T_f): (N, M_0) → {No Fault, Faulty, Uncertain}* as follows:

1) Δ *(s, T_f) = No Faulty:* If *(s) ⊭ R'*, if the first inequality R is satisfied and the second R´ is not satisfied it means that the sequence of events contains no faults.
2) Δ *(s, T_f) = Faulty: If (s) ⊭ R*,the diagnosis state is declared faulty if the R inequality is not satisfied and the R′ inequality is satisfied.
3) Δ *(s, T_f) = Uncertain: If (s) ⊨ R and (s) ⊨ R′*,if the both inequalities R and R' are satisfied, the diagnosis will be uncertain because there is no certainty if the fault has occurred.
4) Δ *(s, T_f) = Impossible: If (s) ⊭ R and (s) ⊭ R′*,if the two inequalities are not satisfied the state of the diagnosis is impossible.

Table 3: Diagnose States

$s = pi(\sigma)$	$(s) \vDash R_1$	$(s) \vDash R'_1$	$\Delta(s, T_f)$
$\mathcal{E}$	Yes	No	*No Fault*
t_1	Yes	No	*No Fault*
t_1, t_3	Yes	No	*No Fault*
t_1, t_5	Yes	Yes	*Uncertain*
t_1, t_9	Yes	No	*No Fault*
$t_1, t_{10}, t_{11}, t_{12}$	Yes	No	*No Fault*
$s = pi(\sigma)$	$(s) \vDash R_2$	$(s) \vDash R'_2$	$\Delta(s, T_f)$
$\mathcal{E}$	Yes	No	*No Fault*
t_1	Yes	Yes	*Uncertain*
t_1, t_3	Yes	No	*No Fault*
t_1, t_5	Yes	No	*No Fault*
t_1, t_9	No	Yes	*Faulty*
$t_1, t_{10}, t_{11}, t_{12}$	Yes	No	*No Fault*

Let s be the sequence of events and $s= p_i(\sigma)$ the corresponding sequence of observable events. Suppose that the diagnoser observes no sequence $(s = \varepsilon)$, which means every variable corresponding to the transition in the set of observable transitions is replaced by zero on the set, R_i, $i \in \{1, 2\}$ R_1, R'_1, R_2, R'_2,by observing we find that both R_i satisfies and the R_i' does not satisfy, hence, we are confident that no fault has happened, we have (s, T_f) = *No Fault*, when $s= t_1$, we have $\#(t_1,s) = 1$ and all other variables zero, by replacing the value of variables on the set, we find that both set R_2 and R'_2 satisfy, thus the diagnose state is (s, T^2_f) = *Uncertain* and (s, T^1_f) = *No Fault*, the sequence $s = t_1,t_3$ all the first set Satisfy R_i and second set R_i'does not satisfy, hence there is no presence of fault, the sequence $s = t_1,t_5$is (s, T^1_f) = *Uncertain*and (s, T^2_f)= *No Fault*.

Assume now that the sequence is $s = t_1, t_{10}, t_{11}, t_{12}$ implies $\#(t_1, s) = 1$, $\#(t_{10}, s) = 1$, $\#(t_{11}, s) = 1$, $\#(t_{12}, s) = 1$, using these values in the table, we observe that the first set satisfies and the second does not satisfy, so we are convinced this sequence is free of faults; thus, (s, T_f) =*No Fault*. Suppose, we have $s=t_1, t_9$ by verifying these values against the reduced sets; we notice that the first R_2 does not satisfy and the second R'_2,satisfy does it; this sequence is Faulty, therefore (s, T^2_f) = *Faulty*. Now, if observing or considering at the same time all the variable corresponding to observable transition set to be 1, the diagnose state is impossible as both reduced sets does not satisfy.

V. CONCLUSION

This paper proposed faults diagnosis modeled partially observable discrete event systems in Petri nets, where the faults were in unobservable transition. We considered two techniques of fault diagnosis, offline and online. In the offline method, the fault diagnosis was based on the elimination method called integer Fourier-motzkin method to eliminate the variable corresponding to unobservable transitions to get two sets of inequalities from the state equation of the modeled Petri nets, then get two reduced sets by adding the original set of inequalities sets to their constraints and the negation, the constraints correspond to the fault's transitions straightwardly. In the online method, the state of diagnosis was checked from the reduced set of inequalities in the variable representing the occurrence of observable events and computed the system behavior status: no fault, faulty, and uncertain. The future investigation will focus on the faults that are not modeled as events.

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