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Cooperative Time-Varying Formation Trajectory Tracking Control for Multiple Heterogeneous Uncertain auvs with Unknown Disturbances

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ABSTRACT: *This paper presents a multi-heterogeneous Autonomous Underwater Vehicle (AUV) formation control method based on a distributed time-varying formation observer and trajectory tracking controller. By introducing a virtual pilot structure, the method effectively coordinates heterogeneous AUV formations and achieves trajectory tracking, enhancing the system's robustness and real-time performance in complex marine environments. To improve robustness against uncertainties and external disturbances, an integral sliding mode control (ISMC) strategy is adopted, with an adaptive disturbance observer (ADO) introduced to estimate and compensate for unmodeled dynamic disturbances. To address the chattering issue in traditional ISMC, a fuzzy integral sliding mode control (FISMC) approach is developed, providing optimal thrust control input under time-varying disturbances and improving AUV trajectory tracking performance. The Lyapunov stability theory proves the input-state stability of the closed-loop system. Simulation results validate the effectiveness of the proposed control protocol, demonstrating the method's capability to achieve the desired formation shape, track predefined trajectories, and maintain smooth control input without high-frequency chattering.*

KEYWORDS*-Multi-heterogeneous AUV formation, time-varying formation observer, trajectory tracking, adaptive disturbance observer.*

I. INTRODUCTION

With the rapid advancement of autonomous technologies, Autonomous Underwater Vehicles (AUVs) have become increasingly vital for a broad spectrum of marine operations,such as seafloor mapping, resource exploration and bathymetric survey, etc.[1],[2],[3],[4]. Cooperative operations among multiple AUVs, as opposed to a solitary AUV, significantly enhance their operational range, flexibility, and overall effectiveness[5].In recent times, there has been a growing focus on the distributed control and coordination of multiple marine vehicles[6].Formation control, known for its diverse applications in cooperative localization, surveillance, and target enclosure, is a rapidly growing field within the domain of distributed control problems. This importance has spurred extensive research in control and robotic systems[7][8]. The main goals of formation control involve modifying the positions, orientations, and velocities of agents while simultaneously preserving the desired shape of the multiagent system.

The challenge of controlling multi-AUV formations is widely recognized, primarily due to the intricate nonlinear dynamics and complex interactivity among multiple autonomous underwater vehicles (AUVs). This difficulty is compounded by the unpredictable, dynamic, and often hostile conditions found in underwater settings.Despite these challenges, the widespread use of AUVs in modern ocean industries has led to a growing interest in multi-AUV formation control from both marine technology and control engineering fields.Various formation control approaches have been reported in the literature, such as the leader-following approach[9], the behavioral approach[10], the artificial potential-based approach[11], and the virtual structure approach[12].In the approach of leader-follower, one or several AUVs are designated as leaders, while others are designed to follow. The strategy of leader-follower is characterized by its simplicity, but the follower AUVs often lack intercommunication capability, resulting in reduced stability and robustness of the system. In case of failure of the leader AUV, it leads to failure of the entire formation[13].In the approach of behavioral, multiple desired behaviors are formulated for each AUV. Each behavior serves a specific purpose, such as reaching the target, avoiding stationary obstacles, evading other AUVs, and maintaining formation[14]. The exchange of information between AUVs is minimal when employing behavior-based strategies. However, there are challenges in implementing fundamental behavior design and local control planning which may impact the overall stability of multi-AUV formation control.In the approach based on artificial potential, collision avoidance is achieved by creating a function that represents a field of forces. The desired position is considered as an attractive force field, pulling the AUV towards it. On the other hand, obstacles are treated as repulsive force fields, pushing the AUV away from them. By calculating and adjusting the resultant force field, the AUV can effectively plan and control its path to avoid collisions. However, in scenarios where multiple obstacles or targets are present in close proximity, there is a possibility of encountering local minima in the potential field which may hinder further progress of the AUV.The virtual structure method is an effective approach for accomplishing heterogeneous formation tasks by creating a virtual structure to form a predetermined shape. The control strategy for each vehicle is developed by utilizing the relative information between the vehicle and its reference.

In addition to the coordination strategy, the multi-AUV formation often faces the complex Marine environment, and the motion control performance of AUV needs to be further improved.The trajectory tracking control of multi-AUV formation under the influence of weak communication is proposed based on leader-follower and virtual leader structures[15].The time-varying delay communication synchronization strategy is studied to combat the communication delay problem in formation control[16]. A bionic sliding mode formation control scheme based on distributed control is proposed, which not only eliminates the flutter phenomenon in traditional SMC, but also enhances the robustness to noise[17]. To tackle the chattering in sliding mode control, a novel reaching law was designed, in which an adaptive gain was incorporated based on the variation of the sliding surface[18]. Furthermore, taking into account the unavailable velocity measure as well as reduction of the conservativeness, a equivalent output injection adaptive sliding mode observer based terminal sliding mode control approach was proposed for trajectory tracking of underwater vehicles[19].An event-based scheme is proposed for the under-actuated AUV formation control, and the parameter uncertainty and unknown disturbance in the dynamic model are dealt with[20].In order to improve the control accuracy of AUV, sliding mode control is combined with adaptive disturbance observer to effectively improve the robustness of AUV[21].However, the above methods either solve weak communication scenarios or adopt kinematic or simplified dynamics models for formation control. Considering many different AUV formation control scenarios, a completely distributed method is proposed to realize heterogeneous AUV formation[22].In order to reduce communication frequency and information exchange between AUVs, a distributed event triggering control technique is proposed for heterogeneous AUV formation[23].Aiming at asynchronous information between heterogeneous AUVs, a synchronization strategy is designed to enable different AUVs to share effective state information and realize cooperative control[24].In order to solve the problem of consistency tracking control for heterogeneous nonlinear multi-agent systems with Bouc-Wen hysteresis input and multi-mode switching, an adaptive fully distributed consistency control protocol is proposed[25].

Nearly all of the cited research used a leader-follower coordination model to design the formation controllers for each vehicle. As previously stated, the effectiveness of this approach relied heavily on the leading vehicle's performance and did not facilitate effective coordination among adjacent vehicles.Therefore, in order to further improve the formation performance, it is quite necessary to adopt the virtual leader-follow structure in this paper.Different from the existing researches, a completely distributed time-varying formation observer is designed for the trajectory tracking control of multi-heterogeneous AUV formation under unknown disturbance. This can not only coordinate heterogeneous AUVs to achieve formation, but also maintain position accuracy between AUVs, and meanwhile it is also assumed that each AUV may suffer from the unknown disturbances either because of hydrodynamic-related phenomena or time-varying marine environments.On the other hand, sliding mode control (SMC) has attracted much attention in the field of trajectory tracking control due to its robustness and simplicity. In order to realize stable trajectory tracking under unknown disturbance, an integral controller is designed. At the same time, in order to solve the buffeting problem in ISMC framework, a new fuzzy integral sliding mode controller (FISMC) is developed by integrating fuzzy control. In addition, in order to estimate and compensate the unknown disturbance, an adaptive disturbance observer (ADO) is introduced. Therefore, a distributed adaptive disturbance observer fuzzy integral sliding mode controller (ADO-FISMC) is developed in this paper to achieve good formation tracking performance.

The contributions of this work can be summarized as follows.

(1)A multi-heterogeneous AUV formation

control method based on distributed time-varying formation observer and trajectory tracking controller is proposed. By introducing a virtual pilot structure, the heterogeneous AUV formation is effectively coordinated and trajectory tracking controlled, thus enhancing the robustness and real-time performance of the system in a complex Marine environment.

(2)A new trajectory tracking controller is designed to improve the robustness of the system to uncertainties and external disturbances by adopting an integral sliding mode control (ISMC) strategy, to further enhance the control effect, an adaptive disturbance observer (ADO) is introduced to estimate and compensate unmodeled dynamics and dynamic disturbances. In order to solve the jitter problem in ISMC, a fuzzy integral sliding mode control (FISMC) method is developed to achieve the optimal thrust control input in the presence of time-varying disturbance and improve the trajectory tracking performance of AUV.

(3)The Lyapunov stability theory proves the input state stability of the closed-loop system the system can maintain stability under bounded disturbance. Simulation experiments verify the effectiveness of the proposed control protocol, and the results show that the proposed method can achieve the target formation shape and track the predetermined trajectory, and the control input is smooth without high-frequency buffeting.

II. BACKGROUND

2.1 Notation

In this work, R denotes the set of real numbers, and R^n is the n-dimensional real space, i.e., vectors of length n with real entries. $R^{m\times n}$ represents the set of all m-by-n real matrices. **0** and **I**, respectively, denote the zero matrix and identity matrix.$|a|$is the absolute value of scalar a. $\|x\|_1$ and $\|x\|_2$ are 1-norm and 2-norm of vector x, respectively. $\|X\|_F$ denotes the Frobenius norm of matrix X. $\xi_{max}(\cdot)$ and $\xi_{min}(\cdot)$ represent the maximum and minimal eigenvalues of a square matrix, respectively. $\otimes$ is the Kronecker product operator, and $Tr(\cdot)$ denotes the trace operator, defined as the sum of the diagonal entries of a square matrix.

2.2Graph Theory

Consider a multiagent system with $N+1$ agents, where N agents are followers (respectively, denoted as node $1,2,\dots,N$) and the remaining one is the leader (denoted as node 0). The communication topology among all agents is represented as a digraph $\mathcal{H}=(\mathcal{U},\mathcal{V})$, where $\mathcal{U}=\{0\}\cup\mathcal{N}$ is a set of $\mathcal{N}+1$ nodes with $\mathcal{N}=\{1,\dots,N\}$, and $\mathcal{V}=\mathcal{U}\times\mathcal{U}$ is a set of edges. The weighted adjacency matrix of digraph $\mathcal{H}$ is represented by $\mathcal{B}=[b_{ij}]$ with $i,j\in\mathcal{U}$ and $b_{ij}=0$. If there is a communication channel from$j$ to node$i$, i.e.,$(j,i)\in\mathcal{V}$,$b_{ij}>0$; otherwise, $b_{ij}=0$. If $b_{i0}=0$, the ith marine vehicle does not receive any information from the leader. A directed graph contains a directed spanning tree if there exists one node, designated as the root, that can reach every other node through a directed path. Further, define a subgraph $\bar{\mathcal{H}}=(\mathcal{N},\bar{\mathcal{V}})$ of digraph $\mathcal{H}$ with $\bar{\mathcal{V}}\subseteq\mathcal{N}\times\mathcal{N}$. The Laplacian of a digraph $\mathcal{H}$ is denoted as $L=[l_{ij}]\in R^{N\times N}$ with $i,j\in\mathcal{N}$, where $l_{ii}=\sum_{j=0}^{N}b_{ij}$ and $l_{ij}=-b_{ij}$ for $i\neq j$.

2.3 Problem Formulation

Consider a group including N heterogeneous AUVs with uncertain nonlinear dynamics, given as follows:

$$\dot{\kappa}_i = \mathcal{J}_i(\kappa_i)\nu_i \#(1)$$

$$M_i\dot{\nu}_i = \pi_i - C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i + \pi_{Ei} \#(2)$$

where i represents the i th AUV with $i \in \mathcal{N}$; $\kappa_i = [x_i, y_i, \varphi_i]$ denotes the ith AUV's position and heading angle in the earth-fixed inertial frame; $\nu_i = [u_i, v_i, r_i]^T$ is the velocity vector in surge(u_i), sway(v_i), and yaw(r_i) in the body-fixed frame;π_i is the control force vector $\pi_i = []$; π_{Ei} denotes the vector of environmental disturbances due to wind, waves, and currents; and $\mathcal{J}_i(\kappa_i)$ is a rotational matrix with

$$\mathcal{J}_i(\kappa_i) = \begin{bmatrix} \cos(\varphi_i) & -\sin(\varphi_i) & 0 \\ \sin(\varphi_i) & \cos(\varphi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that $\mathcal{J}_i(\kappa_i)\mathcal{J}_i^T(\kappa_i) = I$. M_i,$C_i(\nu_i)$, and $D_i(\nu_i)$, respectively, represent the inertia matrix, the centripetal and Coriolis torques, and the damping matrix, where $M_i = M_i^T > 0$.

The following virtual leader dynamics are used to produce a desired tracking reference trajectory:

$$\dot{\kappa}_0 = A\kappa_0 \#(3)$$

where $\kappa_0 \in R^3$ is the state of the leader, and A is a known constant matrix.

Some assumptions and definitions are introduced for the subsequent analyses.

Assumption 1: Parameters of the AUV $M_i, C_i(\nu_i), D_i(\nu_i)$ and π_{Ei} are unknown, and π_{Ei} is bounded with $\|\pi_{Ei}\|_2 \le \bar{\pi}_{Ei}$ but the upper bound π_{Ei} is not given.

Assumption 2: The digraph $\mathcal{H}$ has a directed spanning tree with the leader (7) being root.

For specifying the desired time-varying formation shape, a time-varying vector $\boldsymbol{z} = [z_1^T, z_2^T, \ldots, z_N^T]^T$ is given, where z_i is continuously differentiable, $i \in \mathcal{N}$.

Definition 1: For the multiple AUV systems (3),(4) and the leader (7) on a directed graph $\mathcal{H}$, the time-varying formation tracking is achieved if

$$\lim_{t\to\infty} e_i = 0, \qquad i \in \mathcal{N} \#(4)$$

where signal $e_i = \kappa_i - z_i - \kappa_0$ is regarded as the formation tracking error.

Problem 1: Design a set of distributed control laws π_i for multiple AUV systems (3),(4) subject to uncertainties, such that all AUVs can follow the trajectory of the leader (7) by maintaining an expected time-varying formation pattern, namely, $\lim_{t\to\infty} \kappa_i - z_i - \kappa_0 = 0$, $i \in \mathcal{N}$.

III. CONTROL ARCHITECTURE

In this section, we propose a cooperative control strategy for multiple heterogeneous AUVs that are subject to unknown perturbations. The strategy involves two control steps. Firstly, a completely distributed time-varying formation observer is designed to solve the problem that leader information is not utilized by all followers. This allows followers to collaboratively estimate the leader's message and implement the desired time-varying formation pattern. Secondly, based on the distributed observer design, a distributed fuzzy tracking control scheme using fuzzy integral synovium and adaptive interference observer is proposed. This scheme effectively solves the challenge of multi-AUV facing unknown disturbance and enhances the stability of formation trajectory tracking.

3.1 Distributed Time-Varying Formation Observer Design

A novel adaptive law is proposed for the design of a fully distributed time-varying formation observer ：

$$\begin{cases}\dot{\hat{\kappa}}_i = A\hat{\kappa}_i - s_i(1+\delta_i^T\delta_i)^3\delta_i + \varphi_i \\ \dot{s}_i = \delta_i^T\delta_i, s_i(0) \geq 1 \\ \varphi_i = -Az_i + \dot{z}_i \\ \delta_i = \sum_{j=1}^{N} b_{ij}\left[(\hat{\kappa}_i - z_i) - (\hat{\kappa}_j - z_j)\right] \\ \qquad + b_{i0}(\hat{\kappa}_i - z_i - z_0).\end{cases} \#(12)$$

Theorem 1 : Under Assumption 2, the fully distributed time-varying formation observer (16) can achieve tracking formation guided by the leader (7) specified by $z = [z_1^T, z_2^T, \dots, z_N^T]^T$.

Proof: From observer (16), the error $\epsilon_i = \hat{\kappa}_i - z_i - \kappa_0$ satisfies the following dynamics:

$$\dot{\epsilon}_i = (\mathrm{I} \otimes \mathrm{A} - \mathrm{SHL} \otimes \mathrm{I})\epsilon \#(13)$$

with $h_i(\delta_i^T\delta_i) = (1+\delta_i^T\delta_i)^3$. Define $\bar{\epsilon} = (L \otimes I)\epsilon$. It follows from (17) that

Where $\epsilon = [\epsilon_1^T, \epsilon_2^T, \dots, \epsilon_N^T]^T$, $S = diag\{s_1, s_2, \dots, s_N\}$, and $H = diag\{h_1(\delta_1^T\delta_1), h_2(\delta_2^T\delta_2), \dots, h_N(\delta_N^T\delta_N)\}$

$$\dot{\bar{\epsilon}} = (\mathrm{I} \otimes \mathrm{A} - \mathrm{LSH} \otimes \mathrm{I})\bar{\epsilon} \#(14)$$

Where $\bar{\epsilon} = [\bar{\epsilon}_1^T, \bar{\epsilon}_2^T, \dots, \bar{\epsilon}_N^T]^T$. Based on (12), one obtains that $\delta_i = \bar{\epsilon}_i$ and $\dot{s}_i = \bar{\epsilon}_i^T\bar{\epsilon}_i$.

Choose the following Lyapunov function:

$$U = \sum_{i=1}^{N} s_i k_i \int_0^{\bar{\epsilon}_i^T\bar{\epsilon}_i} h_i(\rho)d\rho + \theta\sum_{i=1}^{N}(s_i - \alpha)^2$$

Where θ and α are two positive constants to be determined. It yields that

$$\dot{U} = 2\sum_{i=1}^{N} s_i k_i h_i(\bar{\epsilon}_i^T\bar{\epsilon}_i)\bar{\epsilon}_i^T\dot{\bar{\epsilon}}_i + \sum_{i=1}^{N}\dot{s}_i k_i\int_0^{\bar{\epsilon}_i^T\bar{\epsilon}_i} h_i(\rho)d\rho + 2\theta\sum_{i=1}^{N}(s_i-\alpha)\dot{s}_i \#(16)$$

Along the trajectory of (14), one has

$$\begin{aligned}\sum_{i=1}^{N} 2s_i k_i h_i(\bar{\epsilon}_i^T\bar{\epsilon}_i)\bar{\epsilon}_i^T\dot{\bar{\epsilon}}_i &= 2\bar{\epsilon}^T(SKH \otimes I)\dot{\bar{\epsilon}} \\ &= 2\bar{\epsilon}^T(SKH \otimes A - SKHLSH \otimes I)\bar{\epsilon} \\ &= \bar{\epsilon}^T[SKH \otimes (A+A^T) - SH(KL+L^TK)SH \otimes I]\bar{\epsilon} \\ &\leq \bar{\epsilon}^T[\xi_3 SKH \otimes I - \xi_2 SHSH \otimes I]\bar{\epsilon} \\ &= \sum_{i=1}^{N}(s_i k_i h_i\xi_3 - s_i^2 h_i^2\xi_2)\bar{\epsilon}_i^T\bar{\epsilon}_i\end{aligned} \#(17)$$

Where $\xi_3 = \xi_{max}(A+A^T)$ and $\xi_2 = \xi_{min}(KL + L^TK) > 0$. For simplicity, $h_i(\bar{\epsilon}_i^T\bar{\epsilon}_i) = (1+\bar{\epsilon}_i^T\bar{\epsilon}_i)^3$ is abbreviated as h_i. It follows that $h_i(\rho) = (1+\rho)^3 \geq 1$ and $h_i(\rho)$ is monotonically increasing. It yields that

$$\begin{aligned}\sum_{i=1}^{N}\dot{s}_i k_i\int_0^{\bar{\epsilon}_i^T\bar{\epsilon}_i} h_i(\rho)d\rho &\leq \sum_{i=1}^{N}\dot{s}_i k_i h_i\bar{\epsilon}_i^T\bar{\epsilon}_i \\ &\leq \sum_{i=1}^{N}\frac{\dot{s}_i k_i^3}{3\beta^2} + \sum_{i=1}^{N}\frac{2}{3}\beta\dot{s}_i h_i^{\frac{3}{2}}(\bar{\epsilon}_i^T\bar{\epsilon}_i)^{\frac{3}{2}} \\ &\leq \sum_{i=1}^{N}\frac{\dot{s}_i k_i^3}{3\beta^2} + \sum_{i=1}^{N}\frac{2}{3}\beta\dot{s}_i h_i^{\frac{3}{2}}(1+\bar{\epsilon}_i^T\bar{\epsilon}_i)^{\frac{3}{2}} \\ &= \sum_{i=1}^{N}\left(\frac{k_i^3}{3\beta^2} + \frac{2}{3}\beta h_i^2\right)\bar{\epsilon}_i^T\bar{\epsilon}_i\end{aligned} \#(18)$$

The initial inequality is derived from the application of the mean value theorem, while the subsequent inequality is obtained by utilizing Young's inequality.

Combining (16)-(18), one obtains

$$\dot{U} \leq \sum_{i=1}^{N}\left(s_i k_i h_i\xi_3 - s_i^2 h_i^2\xi_2 + \frac{k_i^3}{3\beta^2} + \frac{2}{3}\beta h_i^2 + 2\theta s_i - 2\theta\alpha\right)\bar{\epsilon}_i^T\bar{\epsilon}_i. \#(19)$$

Let $\beta = (\xi_2/2)$, $\theta = (1/12)\xi_2$ and $\alpha = (k_i^3/6\beta^2\theta) + \bar{\alpha}$ with $\bar{\alpha} > 0$ being to choose. Since $\dot{s}_i = \delta_i^T\delta_i$ with initial conditions $s_i(0) \geq 1$, one has $s_i \geq 1$. Combining $h_i > 1$, it follows that:

$$-s_i^2 h_i^2 \xi_2 + \frac{2}{3}\beta h_i^2 + 2\theta s_i - 2\theta\alpha + \frac{k_i^3}{3\beta^2}$$
$$\leq -s_i^2 h_i^2 \xi_2 + \frac{2}{3}\beta s_i^2 h_i^2 + 2\theta s_i^2 h_i^2 - 2\theta\bar{\alpha} \quad \#(20)$$
$$= -\frac{1}{2} s_i^2 h_i^2 \xi_2 - 2\theta\bar{\alpha}$$
$$\leq -\frac{1}{\sqrt{3}}\xi_2 s_i h_i \sqrt{\bar{\alpha}}.$$

Then, selecting $\bar{\alpha} > (3k_i^2\xi_3^2/\xi_2^2)$ with (24). One has

$$\dot{U} \leq \sum_{i=1}^{N}\left(s_i k_i h_i \xi_3 - \frac{1}{\sqrt{3}}\xi_2 s_i h_i \sqrt{\bar{\alpha}}\right)\bar{\epsilon}_i^T \bar{\epsilon}_i$$
$$\leq \sum_{i=1}^{N}\left(k_i \xi_3 - \frac{1}{\sqrt{3}}\xi_2 \sqrt{\bar{\alpha}}\right) s_i h_i \bar{\epsilon}_i^T \bar{\epsilon}_i \quad \#(21)$$
$$< 0$$

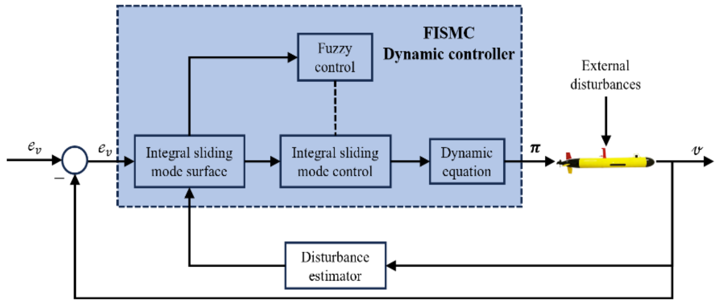
Which shows that system (14) is asymptotically stable. Considering $\bar{\epsilon} = (L \otimes I)\epsilon$ and the nonsingularity of L, one has $\lim_{t\to\infty} \epsilon_i = \lim_{t\to\infty} \hat{\kappa}_i - z_i - \kappa_0 = 0,\ i \in \mathcal{N}$.

The proof is completed.

Remark 2: Observer (5) is static because the coupling gain s is constant, which depends on some global information of the communication topology. It should be pointed out that although the exact value $\xi_2 = \xi_{min}(KL + L^T K) > 0$ is unknown, observer (5) can still operate in a distributed form when s is chosen to be large enough. Compared with observer (5), observer (12) is fully distributed since the coupling gain s_i is updated adaptively based on the local neighbors' information. Parameters φ_i in observer (5) (or (12)) are time-varying formation tracking compensational signals, which are used to compensate the time-varying formation vectors z_i[16].

3.2 Distributed dynamic controller based on ADO-FISMC

To mitigate the impact of hydrodynamic force and unknown dynamics, it is essential to expand the scope of kinematic control to encompass dynamic control. In order to minimize the effects of uncertain disturbances, we employ an ADO-FISMC based dynamic controller that optimizes thrust control signals to enable the AUV to track a predetermined velocity. ISMC offers advantages such as rapid convergence time and robust resistance against disturbances. Additionally, an ADO is devised for estimating dynamic disturbances. To address the issue of chattering in SMC, fuzzy control is implemented to dynamically adjust the switch gain of SMC. The schematic structure depicting the ADO-FISMC based dynamic controller can be observed in Figure 6.



3.2.1 Distributed disturbance observer design

Defines disturbance observer auxiliary variables:

$\dot{\sigma}_i = \hat{M}_i^{-1}(\pi_i - C_i(v_i)v_i - D_i(v_i)v_i - \pi_{Ei} - \hat{\pi}_{w,i})\#(21)$

where, $\hat{\pi}_{wi}$ is the estimate of the systematic error term of the ith AUVπ_{wi}.

The estimate error is defined as:

$$\tilde{\pi}_{w,i} = \pi_{w,i} - \hat{\pi}_{w,i}\#(22)$$

Defines auxiliary sliding mode variable: $\tau_{0,i} = \dot{\sigma}_i - \dot{v}_i$. From the derivative of (21) and (2), the derivative of $\tau_{0,i}$ can be derived as

$$\dot{\tau}_{0,i} = \frac{d}{dt}[\dot{\sigma}_i - \dot{v}_i] = \frac{d}{dt}\left[\hat{M}_i^{-1}(\pi_{w,i} - \hat{\pi}_{w,i})\right] = \hat{M}_i^{-1}(\dot{\pi}_{w,i} - \dot{\hat{\pi}}_{w,i}) + \dot{\hat{M}}_i^{-1}(\pi_{w,i} - \hat{\pi}_{w,i})\#(23)$$

The disturbance observer is designed as follows:

$$\{\dot{\hat{\pi}}_{w,i} = -\hat{M}_i \dot{\hat{M}}_i^{-1}\hat{\pi}_{w,i} + \hat{\alpha}_i \hat{M}_i \mathrm{sgn}(\tau_{0,i})\dot{\hat{\alpha}}_i = \| \tau_{0,i} \| \quad \#(24)$$

If $s_{0,i} = 0$, $\tilde{\pi}_w$ will converge to zero, which means that the disturbance observer can gradually approach the tracking error term.

3.2.2 Design of FISMC

$$\mathcal{S}_i = \left(\frac{d}{dt} + \chi_i\right)^2 \left(\int e_{v,i} dt\right) = \dot{e}_{v,i} + 2\chi_i e_{v,i} + \chi_i^2 \int e_{v,i} dt \#(25)$$

where $\mathcal{S}_i$ is the selected sliding manifold of the ith AUV, $e_{v,i} = v_{r,i} - v_i$ is the velocity error of the ith AUV, and χ_i is a positive definite matrix with constant coefficients.

When the system state reaches the sliding manifold, the first derivative of the sliding surface is expected to be zero:

$$\begin{aligned} \dot{\mathcal{S}}_i &= 0 \\ \ddot{e}_{v,i} + 2\chi_i \dot{e}_{v,i} + \chi_i^2 e_{v,i} &= 0 \quad \#(26) \\ \ddot{e}_{v,i} + 2\chi_i(\dot{v}_{r,i} - \dot{v}_i) + \chi_i^2 e_{v,i} &= 0 \end{aligned}$$

Considering that the dynamic parameters of the AUV system are uncertain, the equivalent control law can be obtained:

$$\pi_{eq,i} = \widehat{M}_i \left(\frac{\ddot{e}_{v,i}}{2\chi_i} + \frac{\chi_i}{2} e_{v,i} + \dot{v}_{r,i}\right) + \hat{C}_i v_i + \widehat{D}_i v_i + \pi_{Ei} + \pi_{w,i} \#(27)$$

To design ISMC, the selected sliding mode surface can be constructed as:

The sign function is replaced by a continuous function to reduce chattering. Therefore, the switching control law π_{sw} is presented as:

$$\pi_{sw,i} = -k_i \tanh(\mathcal{S}_i) \#(28)$$

where, k_i is the switch gain coefficient, $tanh(\mathcal{S}_i)$ is the continuous function.

The ISMC is constructed as:

$$\pi_i = \pi_{eq,i} + \pi_{sw,i} = \widehat{M}_i \left(\dot{v}_{r,i} + \frac{\ddot{e}_{v,i}}{2\chi_i} + \frac{\chi_i}{2} e_{v,i}\right) + \hat{C}_i v_i + \widehat{D}_i v_i + \pi_{Ei} - k_i tanh(\mathcal{S}_i) + \pi_{w,i} \#(29)$$

Lyapunov stability theorem is applied to prove the stability of ISMC. Lyapunov function can be selected as follows:

$$V = \frac{1}{2} \sum_{i=1}^{N} \mathcal{S}_i^T M_i \mathcal{S}_i \#(30)$$

By introducing the derivative of (25), it can be obtained:

$$\dot{V} = \sum_{i=1}^{N} \left(\dot{\mathcal{S}}_i M_i \mathcal{S}_i + \mathcal{S}_i^T \dot{M}_i \mathcal{S}_i + \mathcal{S}_i^T M_i \dot{\mathcal{S}}_i\right) = \sum_{i=1}^{N} \mathcal{S}_i^T \left(-k_i + \pi_{w,i}\right) \#(31)$$

According to the formula above, where $\pi_{w,i} \leq k_i$,$\dot{V} \leq 0$, the control law converges gradually.

In order to further alleviate chattering, a fuzzy controller is adopted to adjust the switch gain of SMC. According to the switching term $k_i tanh(\mathcal{S}_i)$, the sliding mode surface $\mathcal{S}_i$ is set as the input of the fuzzy control and the switching gain k_i is set as the output. The input field is $[-2,2]$ and the output field is $[-1,1]$. According to a large number of different simulations, the fuzzy collection of inputs and output is defined as:

$$\begin{aligned} \mathcal{S}_i &= \{NB\ NM\ NS\ ZE\ PS\ PM\ PB\} \\ k_i &= \{NB\ NM\ NS\ ZE\ PS\ PM\ PB\} \end{aligned} \#(32)$$

And the membership function is formed in:

$$u_i(x_{i,j}) = \exp\left[-\left(\frac{x_{i,j} - \alpha_i}{\sigma_i}\right)^2\right] \#(33)$$

where $x_{i,j}$ is the ith agent's jth input variable of fuzzy rulers, α_i is the i th agent's central value of the membership function, and σ_i is the i th agent's standard deviation.

By using single-valued fuzzier and average fuzzier, the output of the fuzzy controllerError! Reference source not found.is shown as:

$$k_{i,j} = \frac{\sum_{j=1}^{m} \Theta_{k_{i,j}}^{i,j} u_i(\mathcal{S}_{i,j})}{\sum_{j=1}^{m} u_i(\mathcal{S}_{i,j})} \#(34)$$

where $k_{i,j}$ is the output, $\Theta_{k_{i,j}}^{i,j}$ is the output corresponding to the membership value of input, m is the number of fuzzy rules.

IV. SIMULATION

In order to verify the effectiveness of the proposed heterogeneous formation control protocol, simulation experiments are carried out in which four heterogeneous underwater vehicles with controllers are used to track the specified trajectory in a two-dimensional plane, while maintaining the quadrilateral formation mode of the entire AUV group. In addition, practical improvements in control activity smoothness, flutter suppression and interference control have been achieved by conventional synovial control compared with the proposed ADO-FISMC.

The parameters of AUVs are given in the following:

$$M_i = \begin{bmatrix} m_i - X_{\dot{u}_i} & 0 & 0 \\ 0 & m_i - Y_{\dot{v}_i} & m_i x_{g,i} - Y_{\dot{r}_i} \\ 0 & m_i x_{g,i} - N_{\dot{v}_i} & I_{z,i} - N_{\dot{r}_i} \end{bmatrix}$$

$$C_i(v_i) = \begin{bmatrix} 0 & 0 & -(m_i - Y_{\dot{v}_i})v_i - (m_i x_{g,i} - Y_{\dot{r}_i})r_i \\ 0 & 0 & (m_i - Y_{\dot{u}_i})u_i \\ (m_i - Y_{\dot{v}_i})v_i + (m_i x_{g,i} - Y_{\dot{r}_i})r_i & -(m_i - Y_{\dot{u}_i})u_i & 0 \end{bmatrix}$$

$$D_i(v_i) = \begin{bmatrix} -X_{u_i} - X_{|u_i|u_i}|u_i| - X_{u_i u_i u_i} u_i^2 & 0 & 0 \\ 0 & -Y_{v_i} - Y_{|v_i|v_i}|v_i| - Y_{|r_i|v_i}|r_i| & -Y_{r_i} - Y_{|v_i|r_i}|v_i| - Y_{|r_i|r_i}|r_i| \\ 0 & -N_{v_i} - N_{|v_i|v_i}|v_i| - N_{|r_i|v_i}|r_i| & -N_{r_i} - N_{|v_i|r_i}|v_i| - N_{|r_i|r_i}|r_i| \end{bmatrix}$$

where $X_{(\cdot)}, Y_{(\cdot)}$, and $X_{(\cdot)}$ are hydrodynamic parameters according to the notation of [26] and[27].

For the need of simulation, some minor changes are made for different marine vehicles' parameters based on works[28],[29], shown in Table 1 with $Y_{\dot{r}_i} = Y_{\dot{v}_i} = 0$, $N_{\dot{r}_i} = -1$, $X_{u_i u_i u_i} = -5.8664$, $Y_{|v_i|v_i} = -36.2823$, $Y_{|r_i|v_i} = -8.05$, $Y_{|v_i|r_i} = -0.845$, $Y_{|r_i|r_i} = -3.45$, $Y_{r_i} = 0.1079$, $N_{|v_i|v_i} = 5.0437$, $N_{|r_i|v_i} = 0.13$, $N_{r_i} = -1.9$, $N_{|v_i|r_i} = 0.08$, and $Y_{|r_i|r_i} = -0.75$.

Table 1 Values of Marine Vehicles' Parameters

	AUV1	AUV2	AUV3	AUV4
m_i (kg)	27	20	31	43
$I_{z,i}$ (kgm^2)	1.9	1.5	2.3	3.3
$x_{g,i}$ (kg)	0.056	0.05	0.075	0.136
$X_{\dot{u}_i}$ (kg)	-3	-1.66	-2.3	-4.8
$Y_{\dot{v}_i}$ (kg)	-12	-10	-11	-24

X_{u_i} (kg/s)	-0.837	-0.771	-1	-2.2003
$X_{\|u_i\|u_i}$ (kg/m)	-1.539	-1.04	-1.8	-3.6619
Y_{v_i} (kg/s)	-0.956	-0.72	-1.2371	-2.6133
N_{v_i} (kgm^2/s)	0.1154	0.0856	0.1372	0.3342

The communication topology among four AUVs is depicted in Fig.1. The agent 0 is the virtual leader with the following dynamics:

$$\dot{\eta}_0 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \eta_0$$

The initial conditions of the virtual leader, observers and four AUVs are arbitrarily chosen to be $\eta_0(0) = [0.1,1,0.1]^T$, $\hat{\eta}_1(0) = [-0.6,1.2,0.1]^T$, $\hat{\eta}_2(0) = [-0.2,0.9,0]^T$, $\hat{\eta}_3(0) = [-0.5,1.3,0.15]^T$, $\hat{\eta}_4(0) = [-0.4,0.8,0.1]^T$, $\eta_1(0) = [-0.6,1.2,0.1]^T$, $\eta_2(0) = [-0.2,0.9,0]^T$, $\eta_3(0) = [-0.5,1.3,0.15]^T$, $\eta_4(0) = [-0.4,0.8,0.1]^T$, $v_i(0) = [1,1,1]^T$ with $i = 1,2,3,4$.

The desired time-varying distances of the four AUVs with respect to the leader are

$$d_1 = \begin{bmatrix} 0.2 + 0.1\cos(t) \\ 0.2 + 0.1\cos(t) \\ 0 \end{bmatrix},$$

$$d_2 = \begin{bmatrix} 0.2 + 0.1\cos(t) \\ -0.2 + 0.1\cos(t) \\ 0 \end{bmatrix},$$

$d_{1,i} - \eta_{1,0}$ and $y_i - d_{2,i} - \eta_{2,0}$, which all converge to zero over time. Fig.6 demonstrates that the heading angle ψ_i tracks the expected trajectory $\eta_{3,0}$ and the tracking error $\psi_i - d_{3,i} - \eta_{3,0}$

$$d_3 = \begin{bmatrix} -0.2 + 0.1\cos(t) \\ 0.2 + 0.1\cos(t) \\ 0 \end{bmatrix},$$

$$d_4 = \begin{bmatrix} -0.2 + 0.1\cos(t) \\ -0.2 + 0.1\cos(t) \\ 0 \end{bmatrix}.$$

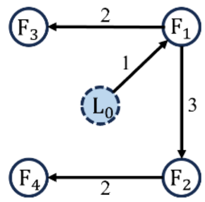


Fig.1. Communication topology $\mathcal{H}$

Figs. 2-7 illustrate the performance of the proposed algorithm. It can be seen from Fig.2 that the trajectory of the leader is circular, and the four heterogeneous AUVs successfully complete time-varying formation starting from different initial positions. Fig.3 presents the position snapshots of the leader and the followers at various timestamps, demonstrating the completion of the formation task. The upper parts of Fig.4 and 5 illustrate the position changes of the agents, while the lower parts show the tracking errors, namely, $x_i -$

approaches zero with time. In addition, Fig.7 displays the convergence of the coupling gains s_1, s_2, s_3, and s_4, taking place around $t = 3s$.

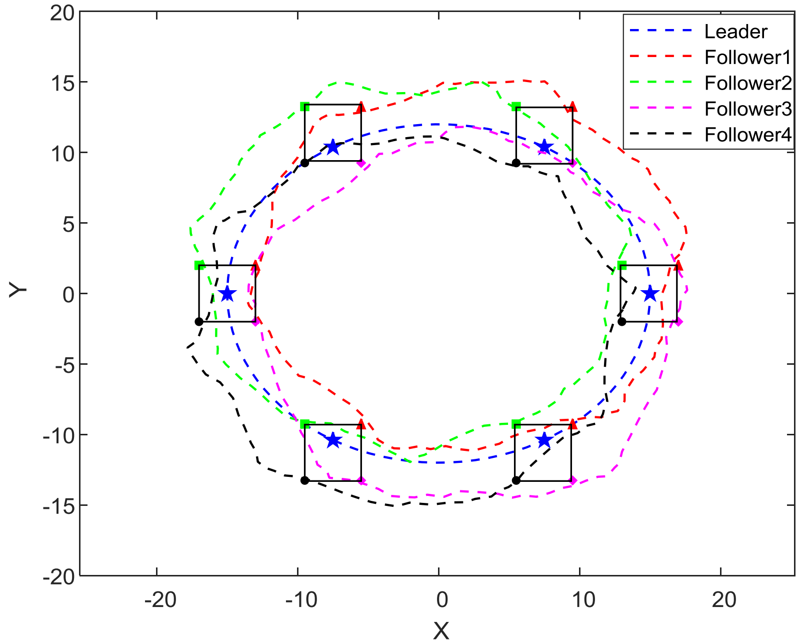


Fig.2. Trajectories of the leader and followers.

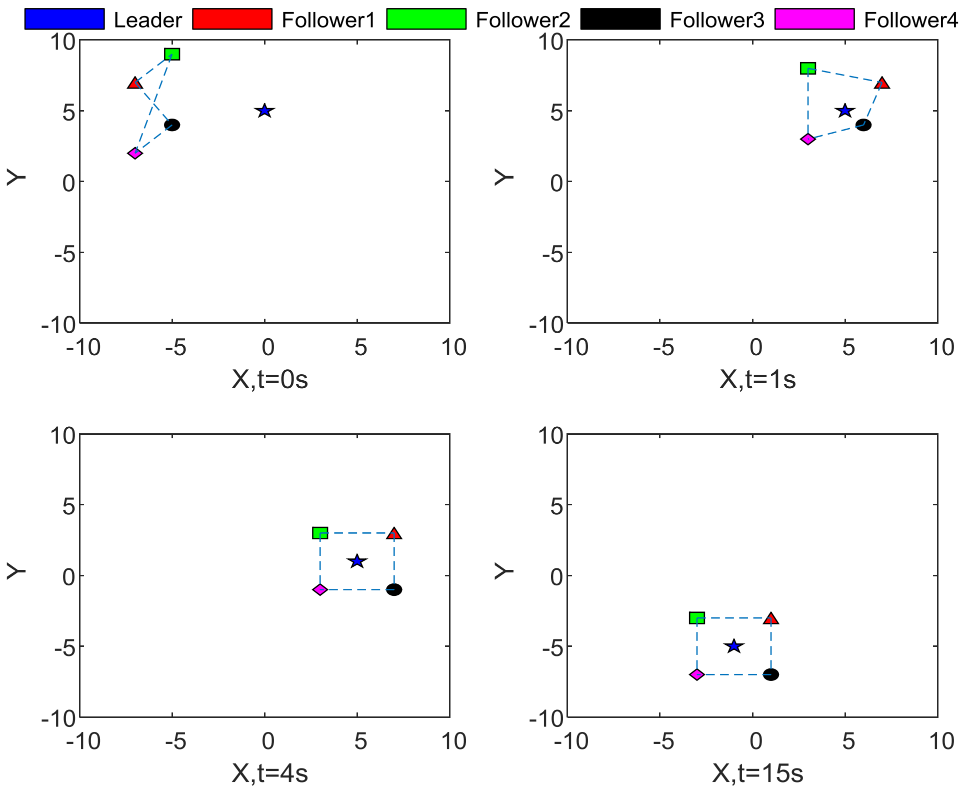


Fig.3. Position snapshots of the leader and followers

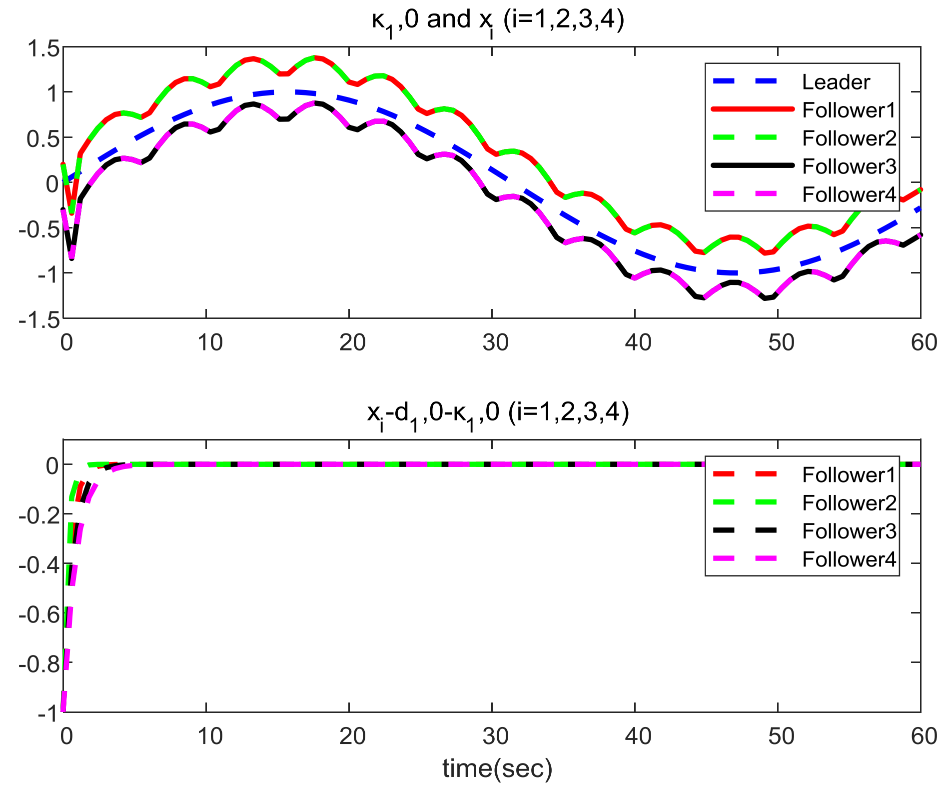


Fig.4. Trajectories of the horizontal position and tracking errors

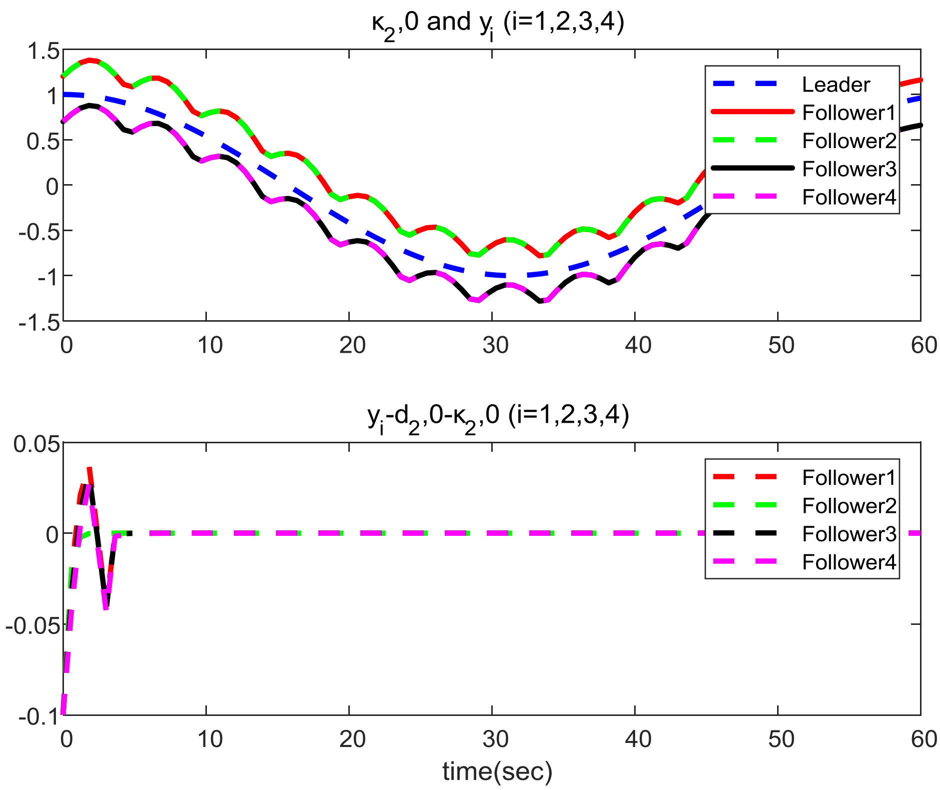


Fig.5. Trajectories of the vertical position and tracking errors

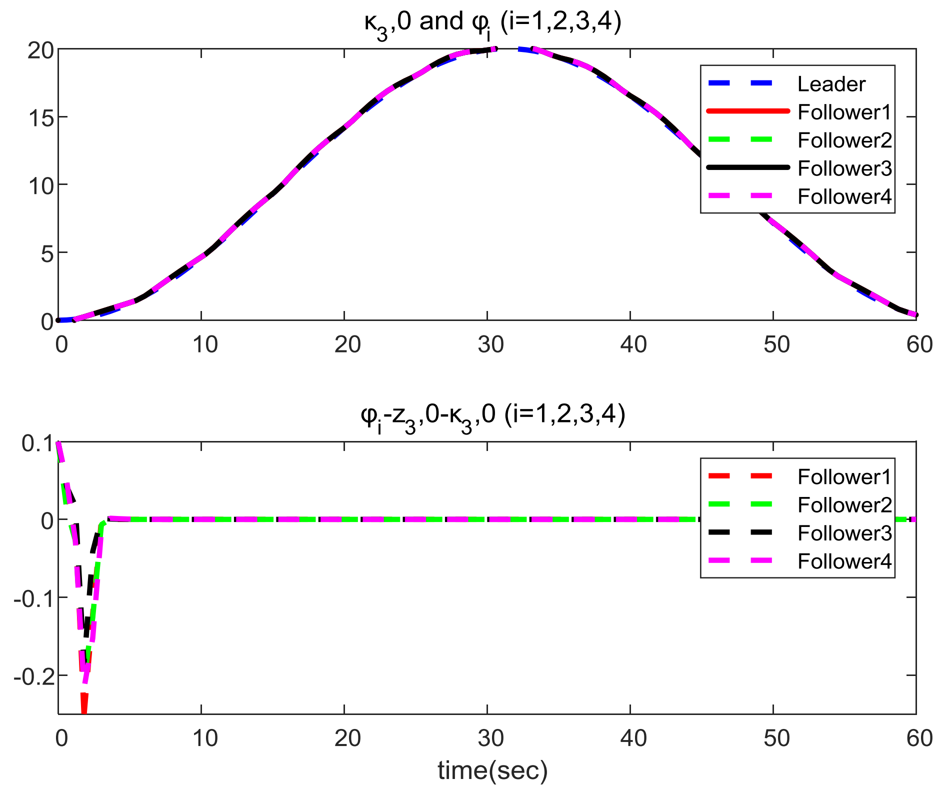


Fig.6. Trajectories of the heading angles and tracking errors.

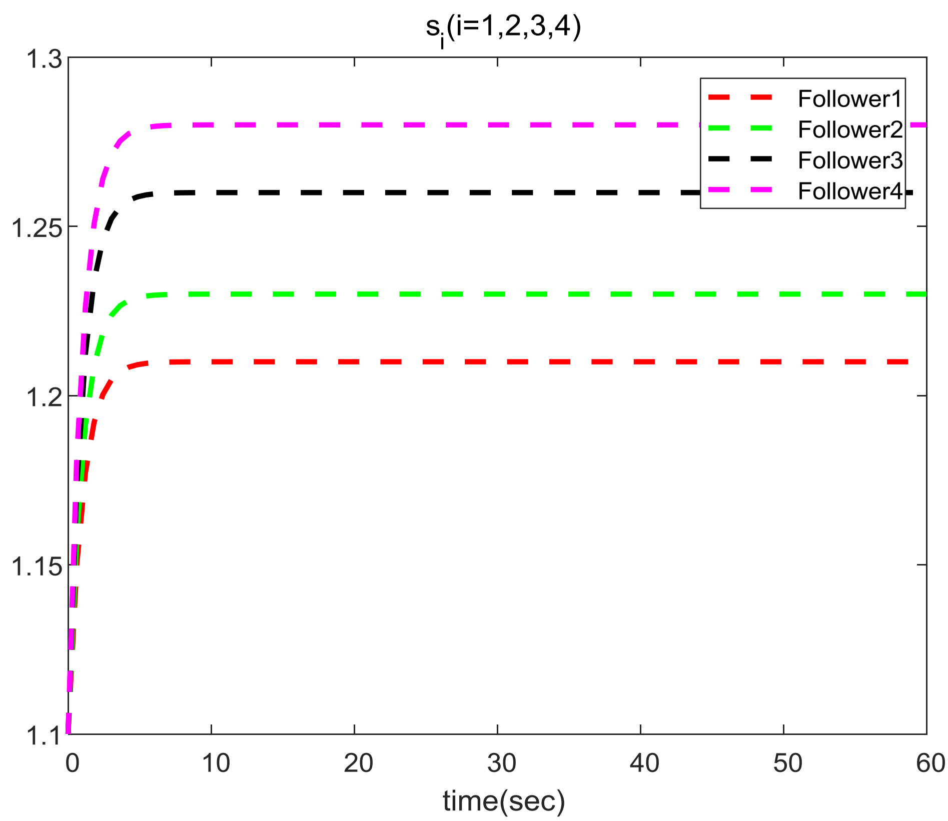


Fig.7. Changes of parameters $\dot{s}_i$ over time, $i \in \{1,2,3,4\}$.

V. CONCLUSION

This paper proposes a cooperative time-varying formation sliding mode control technique for multiple heterogeneous ships subject to unknown nonlinearities and ocean current disturbances. The proposed approach consists of two parts: a distributed time-varying formation observer, which shapes the desired formation pattern and tracks the predefined trajectory, and a decentralized adaptive sliding mode controller that enables the vessels to follow the observer's trajectory. The combination of sliding mode control and adaptive techniques effectively handles uncertainties and actuator failures. The asymptotic stability of the tracking error is proven using the Lyapunov function. Finally, simulation results demonstrate the effectiveness of the proposed approach, showing that cooperative time-varying formation, consensus, and time-invariant formation control all perform as expected.

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